Alan J. Tepley, and Thomas T. Veblen. 2014. Spatiotemporal fire dynamics in mixed-conifer and aspen forests in the San Juan Mountains of southwestern Colorado, USA. *Ecological Monographs* VOL: pp–pp.

APPENDIX B. Derivation of fire-severity metrics from age-structure data

OVERVIEW

To address the challenge of estimating the severity of historical fires in the face of limited data on pre-fire stand structure, we developed fire-severity metrics based on a function, $d_i(n_i)$, describing the rate at which the density of trees that predate (i.e., established before and survived to the present) a given number of fires in a stand decreases with the number of fires experienced. This function is compared to the same function averaged across all stands of both study areas, $\bar{d}(n)$, to estimate the cumulative severity, $S(n_i)$, over the *n* most recent fires per stand. Then, to estimate the severity of each fire experienced by a given stand, $\Delta S(n_i)$, we calculate the proportional change in the cumulative severity metric with each fire added when extending the fire record back in time.

Calculation of the fire-severity metrics requires (1) age-structure data from a large number of stands where most stands include multiple fires, and the full dataset includes a broad range of fire severities; (2) detailed fire history for each stand; and (3) a field sampling method that enables calculation of the density (trees/ha) of trees that predate each fire. Although similar calculations can be conducted where sampling of a subset of trees per stand enables calculation of the proportion but not the density of trees that predate a fire, the ecological interpretations might be complicated. Depending on the method to select the subset of cored trees, two stands could have similar densities of trees that predate a given fire, but the proportion of sampled trees that predate the fire might differ considerably between the two stands. For instance, if the post-fire cohort was denser in one of the stands, it could lead to sampling proportionally more young trees and fewer older trees than in the other stand where trees that established after the fire were present at a lower density.

CALCULATION OF THE FIRE-SEVERITY METRICS

To calculate the fire-severity metrics, we first number the fires in each sampled stand backward from the present such that $n_i = 1$ represents the most recent fire in the i^{th} stand, $n_i = 2$ represents the second most recent fire, etc. The density (trees/ha) of trees in the i^{th} stand that predate the n^{th} most recent fire to burn the stand is represented by $d_i(n_i)$ (Fig. B1a, b). After $d_i(n_i)$ is determined for each of the *m* stands of the dataset (where *m* represents the total number of sampled stands; *m* = 80 for the present study), a function representing the average density of trees that predate a given number of fires across all sampled stands can be calculated as,

$$\bar{d}(n) = \frac{\sum_{i=1}^{m} d_i(n_i)}{m}. \qquad \text{(B.1)}$$

We then calculate a proxy for the cumulative severity over the *n* most recent fires in a stand by comparing the density of trees that predate the n^{th} most recent fire in a given stand to the average density of trees that predate *n* fires across all stands of the dataset:

$$S(n_i) = \frac{d_i(n_i)}{\bar{d}(n)}. \qquad \text{(B.2)}$$

$S(n_i)$ is a unitless number where values >1 indicate the density of trees that predate the n^{th} fire in the i^{th} stand is greater than the average density that predates *n* fires across all stands of the study area. Values <1 indicate the density of trees that predate the n^{th} most recent fire in the i^{th} stand is less than the average density that predates *n* fires per stand across the study area. We view $S(n_i)$ as a proxy for the cumulative severity over *n* fires because the density of trees that predate the n^{th} most recent fire in a stand ($d_i(n_i)$) represents the trees that survived the n^{th} fire and all subsequent fires to the present. Therefore, a value of $S(n_i)$ <1 could arise because each of the *n* most recent fires in the stand burned at higher than average severity, or if one fire burned at much higher than average severity and the others burned at lower severity.

To generate a metric for the severity of each fire experienced by a stand, we calculate the proportional change to $S(n_i)$ with each fire added when extending the record back in time,

$$\Delta S(n_i) = -1 \times \frac{S(n_i)-S(n_i-1)}{S(n_i-1)}. \qquad \text{(B.3)}$$

The proportional change is multiplied by –1 so that increasing values of $\Delta S(n_i)$ represent increasing fire severity. Values are constrained primarily between –1 and +1, but values slightly below –1 are possible if there is no decrease in the density of trees that predate the n^{th} fire compared to the density that predates the next more recent fire (i.e., if $d_i(n_i) = d_i(n_i - 1)$).

$\Delta S(n_i)$ can be interpreted as a measure of whether the density of trees that predate a given number of fires, $d_i(n_i)$, moves closer to or further away from $\bar{d}(n)$ when the n^{th} fire is added to the record than it was when considering only the *n* – 1 most recent fires in the stand. Positive values, indicating higher than average fire severity, result when $S(n_i) < S(n_i - 1)$, meaning the ratio of trees that predate the n^{th} most recent fire in a stand relative to the average density that predates *n* fires is less than the same ratio when considering only the *n* – 1 most recent fires (i.e., when $d_i(n_i)/\bar{d}(n) < d_i(n_i - 1)/\bar{d}(n - 1)$). Negative values, representing lower than average fire severity, result when adding the n^{th} fire to the record produces an increase in the ratio of the density of trees that predate that fire relative to the average density that predates the n^{th} fire per stand compared to the same ratio when considering only the *n* – 1 most recent fires (i.e., when $d_i(n_i)/\bar{d}(n) > d_i(n_i - 1)/\bar{d}(n - 1)$). Thus, positive or negative values of $\Delta S(n_i)$ could be produced regardless of whether the density of trees that predate the n^{th} most recent fire in the stand ($d_i(n_i)$) is greater than or less than the average density that predates the n^{th} fire per stand across the full dataset ($\bar{d}(n)$).

We modified the calculation of the event-level severity metric slightly for the earliest fire of each stand, when no trees predate the fire (i.e., $d_i(n_i) = 0$). In these cases, calculation of $\Delta S(n_i)$ following Eq. B.2 and B.3 would always produce a value of 1, representing the highest possible fire severity. Some stands may have initiated after a single high-severity fire that left no surviving trees. In other stands, however, the density of old trees may have gradually decreased over successive low- and moderate-severity fires to the point where no trees above a certain age were represented in our transect but a small number of older trees may have been recorded had a larger area been sampled. Therefore, to avoid over-estimating the severity of the earliest fire per stand, we calculated $\Delta S(n_i)$ after assuming one tree in each transect predates the earliest fire. Because an area of 0.2 ha (5, 0.04-ha plots) was sampled per transect, this assumption resulted in a change from $d_i(n_i) = 0$ to $d_i(n_i) = 5$ trees/ha for the earliest fire. This adjustment permitted values of $\Delta S(n_i)$ very close to 1 (maximum of 0.99 in this study) if the density of trees that predate the *n* – 1 fire in the stand ($d_i(n_i - 1)$) was high and there was a large decrease to

$d_i(n_i) = 5$ when adding the n^{th} fire to the record. However, the adjustment also allowed for negative values of $\Delta S(n_i)$, representing relatively low fire severity, when $d_i(n_i - 1)$ was low and there was little or no decrease to $d_i(n_i) = 5$ when adding the n^{th} fire. This adjustment was not included when calculating $\bar{d}(n)$.

SAMPLE CALCULATIONS FOR REPRESENTATIVE STANDS

The function, $d_i(n_i)$, is shown for each of the m = 80 stands sampled in the Williams Creek and Squaretop Mountain study areas in Fig. B1a and b, and the average density of trees that predate the n^{th} most recent fire per stand, $\bar{d}(n)$, was calculated by Eq. B.1. We illustrate each step in calculating the cumulative and event-level severity indices (Eq. B.2 and B.3) for three representative stands in Fig. B2. Then, we overlay the event-level severity values for the three stands on their age-structure and fire-history data in Fig. B3. Note that the function, $\bar{d}(n)$, is strongly exponential for this dataset (Fig. B1c, d). If an exponential form is produced when calculated over large datasets, it may be possible to use the single parameter representing the slope of the function to compare the average fire severity among study areas or regions.

We interpret the stand in the left column of Fig. B2 as having experienced relatively low fire severity in the two most recent fires but higher fire severity earlier in the record. Trees that predate each of the two most recent fires were present at a higher density ($d_i(1)$ = 240 trees/ha and $d_i(2)$ = 190 trees/ha) than the overall average density of trees that predate the first and second most recent fire per stand ($\bar{d}(1)$ = 222 trees/ha and $\bar{d}(2)$ = 119 trees/ha). Also, the ratio $d_i(2)/\bar{d}(2)$ was greater than $d_i(1)/\bar{d}(1)$. Thus, we interpret the stand as having experienced lower than average cumulative severity over the two most recent fires ($S(2) = 1.600$), where the event-level severity of the second most recent fire ($\Delta S(2) = -0.482$) was lower than that of the first ($\Delta S(1) = -0.168$; Fig. B2e).

When we extend the fire record in the stand in the left column of Fig. B2 to include the third fire, the cumulative severity over the three most recent fires is still lower than average ($S(3) = 1.304$; Fig. B2d) because the density of trees that predate the third fire ($d_i(3)$ = 90 trees/ha) remains slightly higher than the overall average density that predates the third most recent fire per stand ($\bar{d}(3)$ = 69 trees/ha; Fig. B2b, c). However, because the ratio $d_i(3)/\bar{d}(3)$ was much lower than $d_i(2)/\bar{d}(2)$ (i.e., when the third fire is considered, the density of trees that predate the n^{th} fire exceeds the average density that predates n fires by a much smaller amount than it did when considering only the second most recent fire in the stand), we interpret the event-level severity of the third most recent fire ($\Delta S(3)$ = 0.185) as relatively high (Fig. B2e). When we consider the fourth fire in the stand, the density of trees that predate the fire ($d_i(4)$ = 10 trees/ha) falls well below the average density that predates the fourth most recent fire per stand ($\bar{d}(4)$ = 41 trees/ha), which leads us to interpret that the stand burned at relatively high severity in the fourth fire ($\Delta S(4) = 0.812$; Fig. B2e).

The stand in the center column of Fig. B2 further illustrates the difference between the cumulative and event-level severity metrics. The density of trees that predate the most recent fire in the stand ($d_i(1)$ = 345 trees/ha) was much higher than the average density that predates the most recent fire per stand ($\bar{d}(1)$ = 222 trees/ha) (Fig. B2b, c). Yet, trees that predate the second fire were present at a density ($d_i(2)$ = 145 trees/ha) only slightly greater than the average density that predates the second most recent fire per stand ($\bar{d}(2)$ = 119 trees/ha). Because $d_i(2) > \bar{d}(2)$, we interpret that the cumulative severity over the two most recent fires in this stand was lower than average (Fig. B2d). However, we interpret the severity of the second fire as relatively high ($\Delta S(2) = 0.213$) because when this fire is considered, the ratio of the density of trees that predate the n^{th} most recent fire to the average density that predates n fires per stand ($d_i(2)/\bar{d}(2) = 1.22$) becomes much smaller than the same ratio when considering only the most recent fire ($d_i(1)/\bar{d}(1) = 1.55$; Fig. B2d). Thus, lower than average cumulative severity over the two most recent fires in this stand resulted from very low severity in the most recent fire ($\Delta S(1) = -0.840$) and relatively high severity in the second fire ($\Delta S(2) = 0.213$; Fig. B2e).

The stand in the right column of Fig. B2 illustrates how the metrics may reveal that a single stand experienced substantial variation in fire severity over time. Over the four most recent fires in the stand, the density of trees that predate the n^{th} fire remained similar to the average density of trees that predate n fires across all stands of the dataset (Fig. B2c, d). This similarity led us to interpret that the stand experienced moderate fire severity ($\Delta S(n_i)$ near 0) in each of the four most recent fires (Fig. B2e). When we extend the record to include the fifth and sixth fires, however, the density of trees that predate each of these fires ($d_i(5)$ = 40 trees/ha and $d_i(6)$ = 35 trees/ha) greatly exceeds the overall average density of trees that predate the same number of fires per stand ($\bar{d}(5)$ = 22 trees/ha and $\bar{d}(6)$ = 12 trees/ha; Fig. B2 b, c). Thus, we interpret the severities of the fifth and sixth fires as relatively low ($\Delta S(5) = -0.852$ and $\Delta S(6) = -0.650$; Fig. B2e). When we consider the seventh fire, the density of trees that predate the n^{th} fire in the stand decreases from well above the average density that predates the sixth fire to slightly below the average density that predates the seventh fire per stand (Fig. B2b, c). Therefore, we interpret that the seventh most recent fire in this stand burned at much higher severity ($\Delta S(7) = 0.691$) than any of the more recent fires (Figs. B2e and B3c).

LIMITATIONS OF THIS APPROACH TO ESTIMATING FIRE SEVERITY

The primary limitations of the fire-severity metrics are (1) difficulty in validating them, and (2) the severity values are scaled relative to distribution of values across the dataset, which may complicate comparisons to severity classes commonly applied to contemporary fires. Validation is a problem for all measures of historical fire severity. Ideally, these metrics could be applied to a landscape where the extent and severity of several overlapping fires are documented. Given the rarity of such opportunities, it may be better to evaluate historical metrics by comparison of several age-structure-based approaches. Although thresholds could be applied in an attempt to match contemporary fire-severity classes, the continuous distributions and relative scales of $S(n_i)$ and $\Delta S(n_i)$ may be best suited for comparing relative fire severity among stands within a study area. The methods described here might not be suitable where disturbances other than fire are a primary mortality agent. Also, these methods cannot be used when the number of fires experienced per stand is unknown (e.g., where fire scars are poorly recorded or preserved and the sampling or analysis methods are unable to account for the lack of fire-scar data).

It may appear that using the ratio of the density of trees that predate a given number of fires within a stand to the average density that predates the same number of fires across all stands (Eq. B.2) as the basis for estimating fire severity could over-estimate fire severity in stands that historically had low tree density due to either frequent, low-severity fire or a harsh environment (e.g., thin, rocky soil). However, this is likely to have little influence on the event-level index ($\Delta S(n_i)$). For example, if the density of trees that predate the most recent fire in a stand ($d_i(1)$) is below the average density that predates the most recent fire per stand ($\bar{d}(1)$), the severity of the second most recent fire in the stand is interpreted as higher than average only if the density of trees that predate the second fire falls farther below the average density that predates the second most recent fire per stand (i.e., if $d_i(2)/\bar{d}(2) < d_i(1)/\bar{d}(1)$). Otherwise, severity of the second fire would be interpreted as lower than average, even if $d_i(2)$ remains below $\bar{d}(2)$.

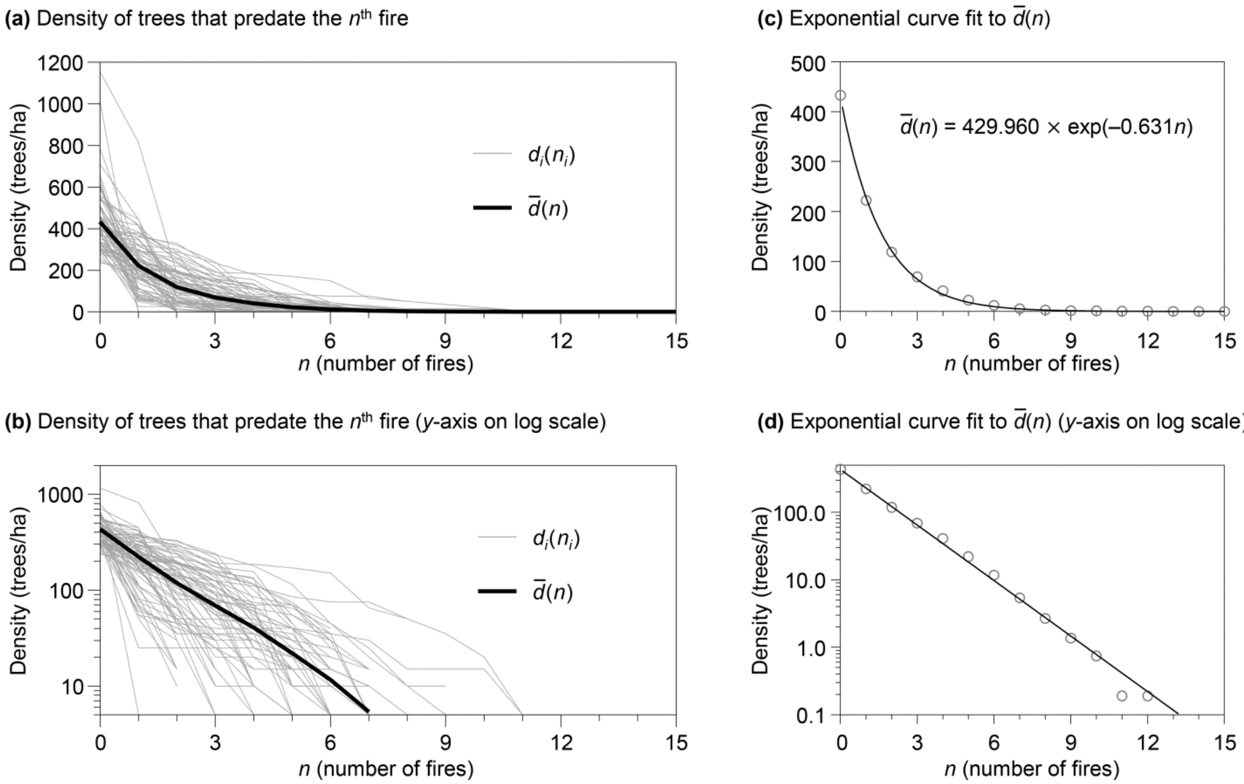

FIG. B1. Comparison of (a) the density of trees (>15 cm dbh) that predate the n^{th} most recent fire per stand ($d_i(n_i)$) for each of the m = 80 stands in the Williams Creek and Squaretop Mountain study areas and the average density of trees that predate a given number of fires across all 80 stands of both study areas ($\bar{d}(n)$). In (b), the same functions are plotted with the y-axis on a log scale. An exponential curve is fit to $\bar{d}(n)$ in (c), and the curve is shown with the y-axis on a log scale in (d). Open circles in (c) and (d) represent values of $\bar{d}(n)$ and the line represents the exponential curve fit to these values ($R^2 = 0.999$, $p<0.001$).

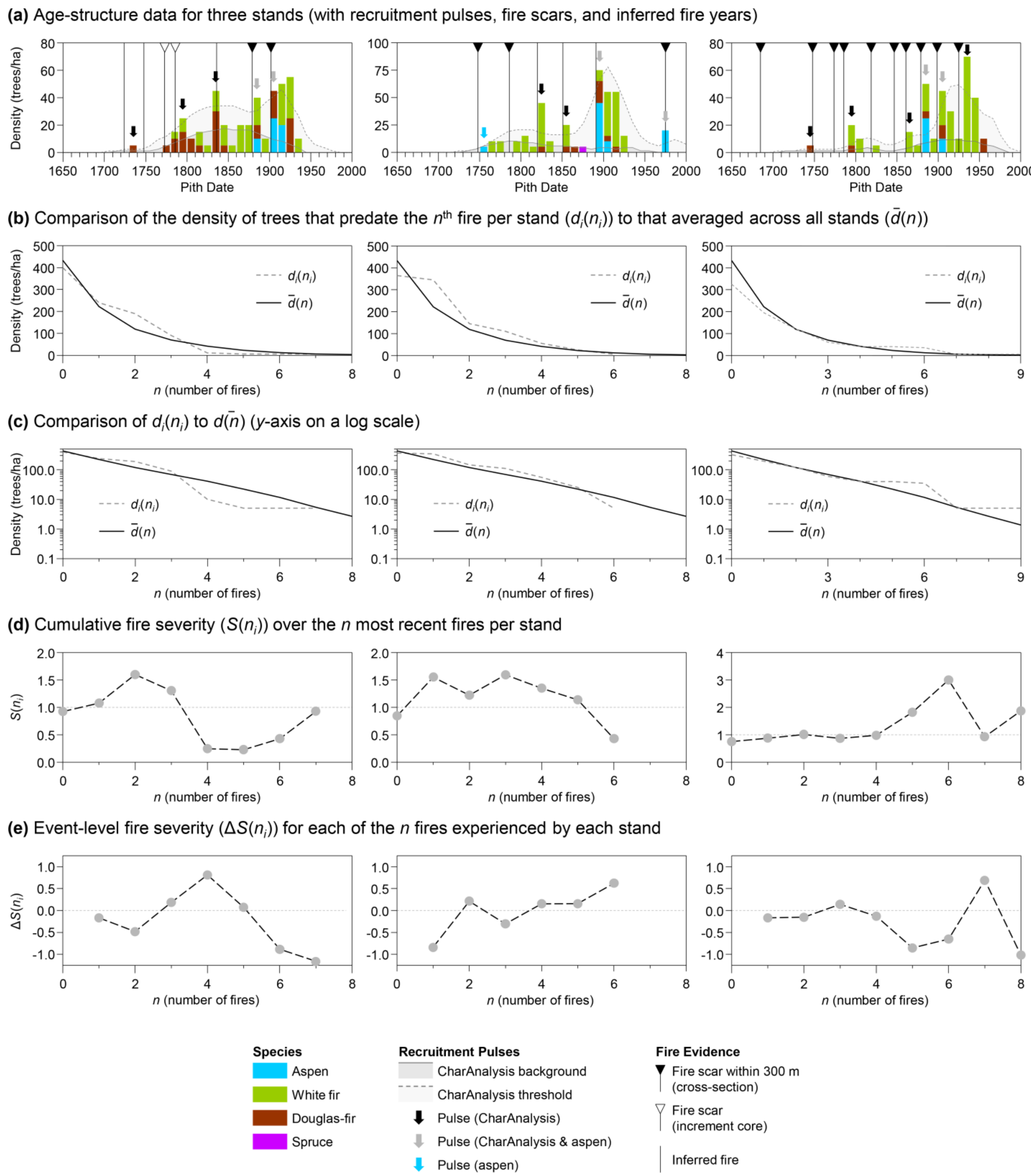

FIG. B2. Illustration of the steps for calculating the fire-severity metrics for three representative stands (each column represents one stand). In (a), age-structure data are presented along with the background and thresholds determined by CharAnalysis (see Appendix A), recruitment pulses, and fire years determined by fire scars (in partial cross-sections or increment cores) or inferred from the spatial distribution of fire scars and recruitment pulses (see Appendix D). Each panel in (b) compares $d_i(n_i)$ to $\bar{d}(n)$, and the same comparisons are made in (c) with the y-axis on a log scale. For each stand, the cumulative severity over the n most recent fires ($S(n_i)$; Eq. B.2) and the severity of each individual fire ($\Delta S(n_i)$; Eq. B.3) are shown in (d) and (e), respectively.

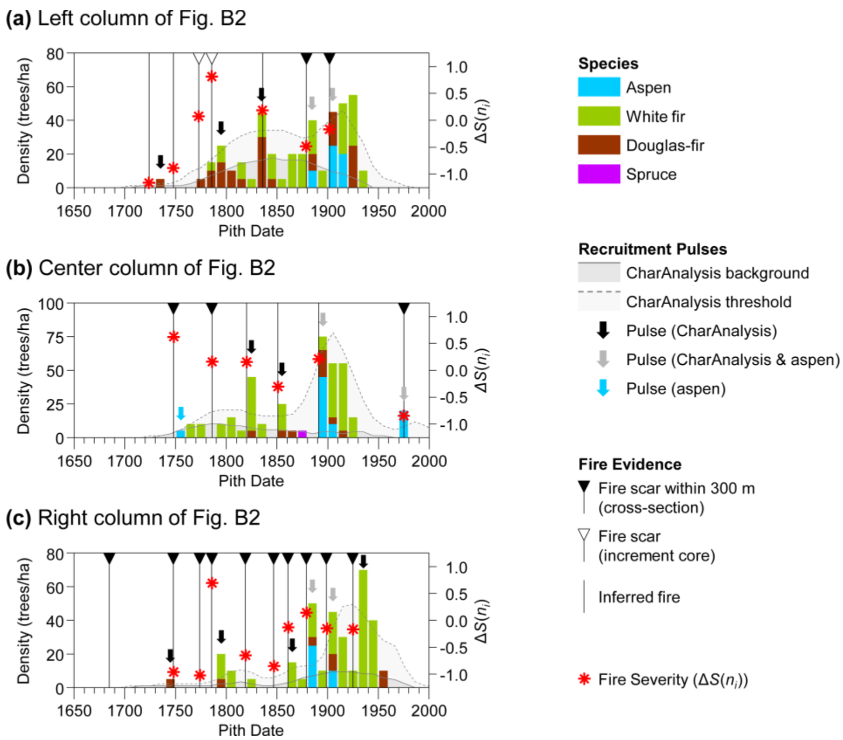

FIG. B3. Comparison of the variation in fire severity over time in relation to age-structure, recruitment pulse, and fire data for the three stands for which the calculation of fire-severity metrics is illustrated in Fig. B2.