Appendix C Expression for the joint and posterior distribution of parameters, latent variables, and data

Combining the process model, the data models, and the parameter models described in Section above provides the full expression for the posterior and joint distributions of states and parameters:

$$
\begin{aligned}
&\left[\mathbf{N}, m, \beta, f_c, f_p, f_n, \psi, \mathbf{p}, v, \boldsymbol{\eta}, \sigma_p^2, p_{sight}, \pi, \rho, \alpha_\pi, \gamma_\pi, \alpha_\rho, \gamma_\rho | \mathbf{Y}_{\text{N.obs}}, \mathbf{Y}_{\text{N.sd}}, \mathbf{Y}_{\text{ratio.calf}}, \mathbf{Y}_{\sigma.\text{calf}}, \mathbf{Y}_{\text{sero}}, \mathbf{Y}_{\text{p.}\mu,}, \mathbf{Y}_{\alpha.\mu}, \mathbf{Y}_{\phi.\mu}\right] \propto \\
&\prod_{t=2}^{41} \text{multivariate normal}\left(\log(\mathbf{n}_{(t)}) | \log(\mathbf{A}_{(t)}\mathbf{n}_{(t-1)}, \mathbf{I}\sigma_p^2\right) \times \\
&\prod_{t=1}^{41} \text{Poisson}\left(y_{\text{N.obs}(t)} | \eta_{(t)} \cdot p_{sight}\right) \text{gamma}\left(\eta_{(t)} \;\middle|\; \frac{\left(\sum_{i=1}^{8} n_{(i,t)}\right)^2}{var_{N.obs}}, \frac{\left(\sum_{i=1}^{8} n_{(i,t)}\right)}{var_{N.obs}}\right) \times \\
&\prod_{t \in \mathbf{y}_{\text{iratio.calf}}} \text{beta}\left(y_{\text{ratio.calf(t)}} \;\middle|\; g\left(\frac{n_{(1,t)} + n_{(4,t)}}{\sum_{i=1}^{8} n_{(i,t)}}, y_{\sigma.\text{calf(t)}}\right)\right) \times \\
&\prod_{t \in y_{\text{iground}}} \text{Dirichlet}\left(\mathbf{y}_{\text{p.}\mu(t)} \;\middle|\; \left[\frac{n_{(8,t)}}{\sum_{i=1}^{8} n_{(i,t)}}, \frac{\left(n_{(3,t)} + n_{(6,t)} + n_{(7,t)}\right)}{\sum_{i=1}^{8} n_{(i,t)}}, \frac{\left(n_{(2,t)} + n_{(5,t)}\right)}{\sum_{i=1}^{8} n_{(i,t)}}, \frac{\left(n_{(1,t)} + n_{(4,t)}\right)}{\sum_{i=1}^{8} n_{(i,t)}}\right] \sum_{j=1}^{4} \alpha_{\mu(i,t)}\right) \times \\
&\prod_{t \in \mathbf{y}_{\text{isero.calf}}} \text{binomial}\left(y_{\text{pos.calf(t)}} \;\middle|\; \frac{\pi n_{(4,t)} + \rho n_{(1,t)}}{n_{(4,t)} + n_{(1,t)}}, y_{\text{n.calf}(t)}\right) \times \\
&\prod_{t \in \mathbf{y}_{\text{isero.yrlg}}} \text{binomial}\left(y_{\text{pos.yrlg(t)}} \;\middle|\; \frac{\pi n_{(5,t)} + \rho n_{(2,t)}}{n_{(5,t)} + n_{(2,t)}}, y_{\text{n.yrlg}(t)}\right) \times \\
&\prod_{t \in \mathbf{y}_{\text{isero.cow}}} \text{binomial}\left(y_{\text{pos.cow(t)}} \;\middle|\; \frac{\pi(n_{(6,t)} + n_{(7,t)}) + \rho n_{(3,t)}}{n_{(6,t)} + n_{(7,t)} + n_{(3,t)}}, y_{\text{n.cow}(t)}\right) \times \\
&\left[\mathbf{n}_{(1)}, m, \beta, f_c, f_p, f_n, \psi, \mathbf{s}, v, \sigma_p^2, p_{sight}, \alpha_\pi, \gamma_\pi, \alpha_\rho, \gamma_\rho\right].
\end{aligned}
$$