Appendix A Description of procedures for estimating sex and age composition of the Yellowstone bison population based on arterial and ground-based classification counts

Here, we describe procedures used to estimate the mean and standard deviations of the proportion of juveniles, yearling females, adult females, and yearling and adult males in the population during years that data were available. These means and standard deviations were used to compose likelihoods for model predictions described in Appendix B. However, the hierarchical models described to estimate sex and age composition from data required hours to compute, so to save computation time for the full model, estimates of population composition were made external to the full model and were entered as data. To preserve consistency in notation, these estimates of composition were written using $\mathbf{y}_{\text{data set name}}$, although strictly speaking, they are parameters that are estimated from data.

We used two types of observations of the sex and age composition of the population, aerial and ground. Aerial observers counted the number of juveniles and older animals in groups observed during early summer. We chose to model the proportion of juveniles within a group as a draw from a beta distribution and sought to estimate the moments of that distribution for each year. We justify this choice because it is unlikely that every group was observed perfectly – there was a estimable probability of over- or under-counting juveniles in each group. Treating the likelihood as a binomial with a single proportion of juveniles common to all groups would overestimate the precision of the estimate, particularly given the large number of animals classified. To be conservative, we chose to model proportion of juveniles in a group as a random variable that could vary with group.

This approach was complicated by bison social structure, which shows strong aggregation in two types of groups distinguished by group size and composition. Large groups were composed of predominantly yearling and adult females with juveniles. Small groups containing no juveniles were dominated by yearling and adult males. To deal with this problem, we estimated the proportion of juveniles in the population as a mixture between large and small groups. We divided the observed groups for each year into two categories indexed by j: groups that contained >= 1 juvenile ($j = 1$) and groups that included 0 juveniles ($j = 2$). The parameters of the beta distribution for each group type for year t $\left(c_{(t,j)}, d_{(t,j)}\right)$ were estimated from

$$\left[\mathbf{p}_{calf(t,j)}, \mathbf{c}_{(t)}, \mathbf{d}_{(t)} | \mathbf{Y}_{\text{calf}(t)}, \mathbf{Y}_{n(t)}\right] \propto \prod_{i=1}^{y_{ng(t,j)}} \text{binomial}\left(y_{\text{calf}(i,t,j)} | p_{calf(i,t,j)}, y_{\text{n}(i,t,j)}\right) \times$$
$$\text{beta}\left(p_{calf(i,t,j)} | c_{(t,j)}, d_{(t,j)}\right) \text{gamma}\left(c_{(t,j)} | .001, .001\right) \text{gamma}\left(d_{(t,j)} | .001, .001\right)$$

where $y_{\text{ng}(t,j)}$ is the number of groups of type j at time t; $y_{\text{n.calf}(i,t,j)}$ is the number of juveniles in group i of type j at time t; $y_{\text{n}(i,t,j)}$ is the total number of animals in group i of type j at time t, and $\mathbf{p}_{calf(t,j)}$ is a $y_{\text{ng}(t,j)}$ element vector of the estimates of the proportion of calves in group type j during year t. The mean of the proportion of juveniles in the population groups of each type was calculated as $\mu_{calf(t,j)} = \frac{c_{(t,j)}}{c_{(t,j)}+d_{(t,j)}}$. By including "calfless" groups in this calculation, we account for uncertainty in the estimate of 0 juveniles in groups. Because the $\mu_{calf(t,j)}$ are functions of random variables, they are also random variables, and we can obtain their posterior distribution using MCMC as described in the section *Parameter estimation*. The average proportion of juveniles in the population at time t $\left(y_{\text{ratio.cal}f(t)}\right)$ was estimated as

$$y_{\text{ratio.calf}(t)} = u_{aerial(t)} \cdot \mu_{(t,1)} + \left(1 - u_{aerial(t)}\right) \cdot \mu_{(t,2)} \qquad \text{(A.1)}$$

where $u_{aerial(t)}$ is the proportion of the population in groups with at least one calf, estimated using

$$u_{aerial(t)} \sim \text{beta}\left(\sum_{i=1}^{y_{n.g(t,1)}} y_{n(i,t,1)} + 1, \sum_{i=1}^{y_{n.g(t,2)}} y_{n(i,t,2)} + 1\right). \tag{A.2}$$

The variance of $y_{\text{ratio.calf}(t)}$ was approximated from the MCMC output (section *Parameter estimation*) as

$$\sigma^2_{y_{\text{ratio.calf}(t)}} = \frac{\sum_{k=1}^{K}\left(y^k_{\text{ratio.calf}(t)} - \frac{\sum_{k=1}^{K} y_{\text{ratio.calf}(t)}}{K}\right)^2}{K} \tag{A.3}$$

where k is the iteration number and K is the total number of iterations.

Ground observations consisted of counts of the number of calves, yearlings, and adult females, and yearling and older males in a sample of groups of bison encountered by observers during July. We chose to model the proportion of each age and sex class within a group as a draw from a Dirichlet distribution and sought to estimate the moments of that distribution hierarchically. As above, we divided the data types of groups indexed by j, j =1 if the group contained 0 >= 1 juvenile and $j = 2$ if the group contained 0 juveniles. The vector of parameters for the distribution of group sex and age compositions $\left(\boldsymbol{\alpha}_{(t,j)}\right)$ was estimated for each type of group during each year using

$$\left[\mathbf{w}_{(t)}, \boldsymbol{\alpha}_{(t)} | \mathbf{y}_{\mathbf{glcass}(t)}, \mathbf{y}_{\mathbf{gn}(t)}\right] \propto \prod_{i=1}^{n_{g(t,j)}} \prod_{j=1}^{2} \text{multinomial}\left(\mathbf{y}_{\mathbf{glcass}(i,t,j)} | \mathbf{w}_{(i,t,j)}, \mathbf{y}_{\mathbf{gn}(i,t,j)}\right) \times \text{Dirichlet}\left(\mathbf{w}_{(i,t,j)} | \boldsymbol{\alpha}_{(t,j)}\right) \prod_{k=1}^{4} \text{gamma}\left(\alpha_{(t,j,k)} | .001, .001\right),$$

$n_{g(t,j)}$ is the number of groups of type j classified in year t; $\mathbf{y}_{\mathbf{gclass}_{(i,t,j)}}$ is the observed number of animals in each sex and age class in group i of type j during year t, $\text{y}_{\text{gn}(i,t,j)}$ is the observed number of individuals in a group i of type j, and $\mathbf{w}_{(i,t,j)}$ is the four element vector of true, unobserved proportions of each age-sex class in group i of type j during year t. To estimate the vector of parameters across the entire population $\left(\alpha_{\mu(t)}\right)$, we used

$$\boldsymbol{\alpha}_{\mu(t)} = u_{ground(t)}\boldsymbol{\alpha}_{(t,1)} + \left(1 - u_{ground(t)}\right)\boldsymbol{\alpha}_{(t,2)} \tag{A.4}$$

where $u_{ground(t)}$ is the proportion of the population in groups with at least one calf:

$$u_{ground(t)} \sim \text{beta}\left(\sum_{i=1}^{n_{g(t,1)}} y_{gn(i,t,1)} + 1, \sum_{i=1}^{n_{g(t,2)}} y_{gn[(i,t,2)} + 1\right). \qquad \text{(A.5)}$$