Appendix A.

Analytic estimators for rarefaction (interpolation) of Hill numbers (${}^{q}\hat{H}$) of order $q = 0$ (first row of equations), $q = 1$ (second row), and $q = 2$ (third row), given a reference sample (Chao et al. 2014). The last row gives equations for sample completeness (coverage, $\hat{C}$) as a function of sample size. In abundance-based rarefaction: n is the reference sample; m is defined as the subsample size for rarefying community; S_{obs} is the observed species richness in n; X_i is a vector of length S_{obs}, the elements of which are the observed abundances; k is the vector of sampling units, from 1 to m; and $\hat{f}_k(m)$ is an unbiased estimator of the number of species detected in exactly m sampling units, where $\hat{f}_k(m) = \sum_{X_i \geq k} \frac{\binom{X_i}{k}\binom{n-X_i}{m-k}}{\binom{n}{m}}$, provided that $m < n$, and $k \geq 1$.

In sample-based rarefaction: T is the reference sample; t is defined as the subsample size for rarefying community; S_{obs} is the observed species richness in T; Y_i is the matrix of incidence data, with $i = 1$ to S_{obs} rows and $j = 1$ to T columns; k is the vector of sampling units, from 1 to t; $\hat{U}_t$ is an unbiased estimator of the expected total number of incidences (U) in t sampling units, $\hat{U}_t = t \times U/T$; and $\hat{Q}_k(m)$ is an unbiased estimator of the number of species detected in exactly k sampling units, where $\hat{Q}_k(t) = \sum_{Y_i \geq k} \frac{\binom{Y_i}{k}\binom{T-Y_i}{t-k}}{\binom{T}{t}}$, provided that $t < T$, and $t \geq 1$. For the reference sample, the observed Hill number of order q is ${}^{q}\hat{D}(n) = {}^{q}D_{obs} = \left[\sum_{Xi \geqslant 1} (X_i/n)^q\right]^{1/(1-q)}$ in abundance-based rarefaction, and ${}^{q}\hat{\triangle}(T) = {}^{q}\triangle_{obs} = \left[\sum_{k=1}^{t} (k/U)^q \times Q_k\right]^{1/(1-q)}$ in incidence-based rarefaction. The coverage of the reference sample is estimated by $\hat{C}(n) = 1 - \frac{f_1}{n}\left[\frac{(n-1)\times f_1}{(n-1)\times f_1 + 2\times f_2}\right]$ in abundance-based rarefaction, dsand $\hat{C}(T) = 1 - \frac{Q_1}{T}\left[\frac{(T-1)\times Q_1}{(T-1)\times Q_1 + 2\times Q_2}\right]$ in incidence-based rarefaction.

	Abundance-based $(m < n)$	Incidence-based $(t < T)$
$q = 0$	$^{0}\hat{H}(m) = \sum_{k=1}^{m} \hat{f}_k(m) = S_{obs} - \sum_{Xi \geqslant 1} \frac{\binom{n-X_i}{m}}{\binom{n}{m}}$	$^{0}\hat{H}(t) = S_{obs} - \sum_{Yi \geqslant 1} \frac{\binom{T-Y_i}{t}}{\binom{T}{t}}$
$q = 1$	$^{1}\hat{H}(m) = exp\left(\sum_{k=1}^{m}\left(-\frac{k}{m} \times log\frac{k}{m}\right) \times \hat{f}_k(m)\right)$	$^{1}\hat{H}(t) = exp\left(\sum_{k=1}^{t}\left(-\frac{k}{\hat{U}_t} \times log\frac{k}{\hat{U}_t}\right) \times \hat{Q}_k(t)\right)$
$q = 2$	$^{2}\hat{H}(m) = \frac{1}{\sum_{k=1}^{m}\left(\frac{k}{m}\right)^2 \times \hat{f}_k(m)}$	$^{2}\hat{H}(t) = \frac{1}{\sum_{k=1}^{m}\left(\frac{k}{\hat{U}_t}\right)^2 \times \hat{Q}_k(t)}$
Coverage	$\hat{C}(m) = 1 - \sum_{Xi \geqslant 1}^{m} \frac{X_i \binom{n-X_i}{m}}{n \binom{n-1}{m}}$	$\hat{C}(t) = 1 - \sum_{Yi \geqslant 1}^{m} \frac{Y_i \binom{T-Y_i}{t}}{n \binom{T-1}{t}}$