Appendix B. Explanation to get the threshold for pupation and the associated parameters.

The threshold for pupation is $[E_R^{\,j}] = s_j [E_R^{\,m}]$

with:

$$[E_R^{\,m}] = (1-\kappa)[E_m] g \frac{\dot{k}_E + \dot{k}_M}{\dot{k}_E - g\dot{k}_M} \qquad \text{(Eq. B.1)}$$

where:

- $[E_m] = \frac{\{\dot{p}_{Am}\}}{\dot{\upsilon}}$ (J/cm^3) is the reserve capacity and represents the ratio of assimilation and mobilization fluxes.

- $g = \frac{[E_G]}{\kappa [E_m]}$ (-) is the energy investment ratio and stands for the cost of new biovolume relative to the maximum potentially available energy for growth plus maintenance.

- $\dot{k}_E$ (d^{-1}) relates to the energy conductance of the embryo $\dot{\upsilon}$ as $\dot{k}_E = \dot{\upsilon} / L_b$

- $\dot{k}_M$ (d^{-1}) is the somatic maintenance rate coefficient and expresses the maintenance cost relative to the cost of structure $\dot{k}_M = \frac{[\dot{p}_M]}{[E_G]}$.

In this appendix we present the computation of $[E_R^{\,m}]$

In DEB theory (Kooijman 2010) the reproduction flux is given by

$$\dot{p}_R = (1-\kappa)\dot{p}_C - \dot{p}_J \qquad \text{(Eq. B.2)}$$

which in specific life stages (adult stage in the standard DEB model, larval and adult stage in the present holometabolous insect DEB model) is accumulated in the reproduction buffer

$$E_R(t) = \int_0^t \dot{p}_R(s)ds \qquad \text{(Eq. B.3)}$$

In equation 3 the integral starts from 0 that represents the time when the accumulation starts.

We know that (see Kooijman 2010):

$$\dot{p}_C = E(\dot{k}_E - \dot{r}) = E\frac{[E_G]\dot{k}_E + [\dot{p}_M]}{\kappa[E] + [E_G]} \qquad \text{(Eq. B.4)}$$

Using equations 2 and 4 one can rewrite equation 3

$$E_R(t) = \int_0^t ((1-\kappa)E(\dot{k}_E - \dot{r}) - \dot{p}_J)ds \qquad \text{(Eq. B.5)}$$

Under constant feeding conditions $f = \frac{[E]}{[E_m]}$ which is the same as:

$$E = [E_m]fV \text{ (Eq. B.6)}$$

Replacing equation 6 in equation 5 and knowing that $\dot{p}_J$ is constant during the larval period (see Table 4) we have:

$$E_R(t) = (1-\kappa)(\dot{k}_E - \dot{r})[E_m]f\int_0^t Vds - t\dot{p}_J \qquad \text{(Eq. B.7)}$$

During the larval phase the organism behaves as a V1-morph, meaning that at constant feeding conditions, i.e. constant f , the growth is exponential:

$$V(t) = V_b e^{\dot{r}t} \quad \text{(Eq. B.8)}$$

and the reproduction buffer E_R is then:

$$E_R(t) = (1-\kappa)[E_m] f V b(\dot{k}_E / \dot{r} - 1)(e^{\dot{r}t} - 1) - t\dot{p}_J \quad \text{(Eq. B.9)}$$

The trigger for pupation is linked to the reproduction buffer per unit of volume $[E_R] = \frac{E_R}{V}$ with V given by equation 8 and E_R given by equation 9. The onset of pupation occurs therefore when E_R reaches a proportion s_j of the possible maximum value of $[E_R]$. The maximum value that $[E_R]$ can take is

$$[E_R{}^m] = (1-\kappa)[E_m] f(\dot{k}_E / \dot{r} - 1) \qquad \text{(Eq. B.10)}$$

From equation 4 one can compute the growth rate

$$\dot{r} = \frac{\dot{k}_E g^{-1} f - \dot{k}_M}{1 + g^{-1} f} \quad \text{(Eq. B.11)}$$

And the maximum reproduction buffer per unit of volume $[E_R]$ for $f = 1$ is the expression shown in the beginning of the appendix:

$$[E_R{}^m] = (1-\kappa)[E_m] g \frac{\dot{k}_E + \dot{k}_M}{\dot{k}_E - g\dot{k}_M} \quad \text{(Eq. B.1)}$$