Appendix B: Stability of the null model

The Jacobian matrix, which consists of partial derivatives of the differential equations, is:

$$\mathbf{J} = \begin{pmatrix} -(h+r) & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -cy_i & 0 & \cdots & c(1-x_\phi)-h-2cy_i & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \qquad \textbf{(B.1)}$$

Because the determinant for this matrix is just the product of the diagonal entries, the eigenvalues of the Jacobian are described by

$$0 = \left[-(h+r)-\lambda\right]\Pi_{i=1}^{S}\left[c(1-x_\phi)-h-2cy_i-\lambda\right]. \qquad \textbf{(B.2)}$$

The y_is are equivalent for all i, so we can simplify to

$$0 = \left[-(h+r)-\lambda\right]\left[c(1-x_\phi)-h-2cy_i-\lambda\right]^S. \qquad \textbf{(B.3)}$$

λ thus can take two values, $-(r+h)$ or $c(1-x_\phi)-h-2cy_i$. The first is always negative, since r and h are non-negative. When $y_i = 0$, $\lambda = \left(\frac{cr}{r+h}-h\right)$ is negative (and thus this solution is locally stable) whenever $c < h(r+h)/r$. The other solution is $y_i = r/(r+h) - h/c$, at which the dominant eigenvalue is

$$\lambda = c\frac{r}{r+h}-h-2c\left(\frac{r}{r+h}-\frac{h}{c}\right). \qquad \textbf{(B.4)}$$

Setting $\lambda < 0$, we find that this solution is stable when $c > h(r+h)/r$.