Appendix A: Derivation of the mass balance constraint and of zero-net-growth isoclines.

The model consists of a system of three ordinary differential equations describing the rates of change of grazer biomass density G, algal biomass density R, and mineral nutrient concentration N (see parameter definitions in Table 1):

$$\frac{dG}{dt} = e\frac{aR^b}{1+ahR^b}G - mG - DG \tag{A.1a}$$

$$\frac{dR}{dt} = r\frac{NR}{N+N_H} - \frac{aR^b}{1+ahR^b}G - DR \tag{A.2a}$$

$$\frac{dN}{dt} = DN_{in} - DN - Q_R r\frac{NR}{N+N_H} + (Q_R - Q_G e)\frac{aR^b}{1+ahR^b}G + Q_G mG \tag{A.3}$$

The grazer and algae equations are described in the main text of the manuscript. In the nutrient equation the five terms describe, respectively, the rates at which mineral nutrients enter and leave the system, are taken up by growing algae, and are recycled from algal nutrients ingested but not assimilated by grazers and from grazer losses. Grazer and algal biomass dynamics can be expressed in units of nutrient:

$$Q_G\frac{dG}{dt} = Q_G e\frac{aR^b}{1+ahR^b}G - Q_G mG - Q_G DG \tag{A.1b}$$

$$Q_R\frac{dR}{dt} = Q_R r\frac{NR}{N+N_H} - Q_R\frac{aR^b}{1+ahR^b}G - Q_R DR \tag{A.2b}$$

The dynamics of total nutrients in the system N_{tot} equals the sum of eqs. (A.1b), (A.2b) and (A.3):

$$\frac{dN_{tot}}{dt} = \frac{d(N + Q_R R + Q_G G)}{dt} = D(N_{in} - N - Q_R R - Q_G G) = D(N_{in} - N_{tot}) \tag{A.4}$$

Total nutrients are in equilibrium $\left(N_{tot}/dt = 0\right)$ when $N_{tot} = N_{in}$. Eq. A.4 can thus be rewritten as

$$\frac{dN_{tot}}{dt} = DN_{in} - D\left(N_{in} + n_{tot}\right) = -Dn_{tot} = \frac{dn_{tot}}{dt}, \tag{A.5}$$

where $n_{tot} = N_{tot} - N_{in}$ is an initial displacement of total nutrients from equilibrium. Thus, any initial displacement n_{tot} from equilibrium decays to zero at rate $-Dn_{tot}$ and total nutrients approach

$$N_{tot} = N + Q_R R + Q_G G = N_{in}. \tag{A.6}$$

Eq. (A.6) shows that the system is asymptotically mass balanced, i.e. the sum of nutrients in the algal, grazer and mineral nutrient compartments equals the nutrient supply concentration N_{in} as time goes to infinity.

The algal isocline can be derived by rearranging eq. (A.6) and substituting the resulting expression $N = N_{in} - Q_R R - Q_G G$ into eq. (A.2a) at equilibrium:

$$\frac{dR}{dt} = 0 = r\frac{(N_{in} - Q_R R - Q_G G)R}{N_{in} - Q_R R - Q_G G + N_H} - \frac{aR^b}{1 + ahR^b}G - DR \tag{A.7}$$

Solving eq. (A.7) for G yields two solutions for the algal isocline

$$G_{1,2} = \frac{1}{2}\frac{r\left(1 + ahR^b\right)Q_G + \left(N_{in} - Q_R R + N_H\right)aR^{b-1} - D\left(1 + ahR^b\right)Q_G}{Q_G aR^{b-1}} \pm \sqrt{\frac{\left[r\left(1 + ahR^b\right)Q_G + \left(N_{in} - Q_R R + N_H\right)aR^{b-1} - D\left(1 + ahR^b\right)Q_G\right]^2}{4\left(Q_G aR^{b-1}\right)^2} - \frac{\left[(r - D)\left(N_{in} - Q_R R\right) - DN_H\right]\left(1 + ahR^b\right)}{Q_G aR^{b-1}}} \tag{A.8}$$

Only one of the two solutions in eq. (A.8) is biologically meaningful, i.e. only subtraction of the root fulfills the mass balance constraint $G \le (N_{in} - Q_R R)/Q_G$.

The grazer isocline can be found by solving eq. (A.1a) for R at equilibrium:

$$\frac{dG}{dt} = 0 = e\frac{aR^b}{1+ahR^b}G - mG - DG \tag{A.9}$$

which yields

$$R = \sqrt[b]{\frac{(m+D)}{a(e-(m+D)h)}} \tag{A.10}$$