Appendix B – Details of the univariate and bivariate wavelet analyses.

Throughout this study, we relied on wavelet analysis to detect periodic behavior in single time series, and time-varying associations between time series. In this appendix we briefly describe our procedures following the terminology of Cazelles et al. (2007) and the Matlab package based on that work. For a complete overview of wavelet analysis, please see Torrence and Compo (1998), Cazelles et al. (2007, 2008), Grinsted et al. (2004), or one of the many other papers focused on introducing wavelet analysis.

The continuous wavelet transform is similar to Fourier analysis in that it breaks down a time series into its cyclical components but has the additional feature of characterizing temporal variation in periodicity (Torrence and Compo 1998). The continuous wavelet transform $W_x(a,\tau)$ is the convolution of the time series $x(t)$ with a wavelet function $\psi_{a,\tau}$, which is localized in both the time and frequency domains. Parameter a modifies the scale of the wavelet function, and τ represents the point in the time series. By repeating the wavelet transform for all $\tau = t$ and scales of interest, we can derive the wavelet power spectrum $S_x(a,\tau) = \left\| W_x(a,\tau) \right\|^2$ to detect periodic behavior. If the wavelet function is complex, the phase angle can be calculated as well. The Matlab package implements the widely-used, complex Morlet wavelet $\psi(t) = \pi^{-1/4} \exp\left(-i2\omega_0 t\right) \exp\left(-t^2/2\right)$, where constant $\omega_0 = 6$ controls number of oscillations within the wavelet function. We modified the Matlab package to correct a bias towards low-frequency signals inherent to traditional wavelet analysis (Liu et al. 2007). While the wavelet transform does not require a specific distribution of $x(t)$, distributions that are far from normal may produce less reliable results (Jevrejeva et al. 2003, Grinsted et al. 2004). Specifically, the Morlet wavelet is normally distributed so transformations to this distribution may be appropriate (T. Compo, *personal communication*). Therefore, we transformed time series to approximate a normal distribution.

The cross-wavelet spectrum and wavelet coherence are bivariate extensions of the continuous wavelet transform that can detect associations among time series that may vary through time (Grinsted et al. 2004, Cazelles et al. 2008). The cross-wavelet spectrum between time series $x(t)$ and $y(t)$ is defined as $W_{xy}(a,\tau) = W_x(a,\tau) W_y^*(a,\tau)$, where * indicates the complex conjugate. The cross-wavelet spectrum indicates common areas of power between W_x and W_y., and with a complex wavelet can also derived the phase difference between the two time series. Wavelet coherence is $W_{x,y}$ normalized by each wavelet spectrum, given as

$$R_{x,y}(a,\tau) = \frac{\left\| \left\langle W_{x,y}(a,\tau) \right\rangle \right\|}{\left\| \left\langle W_{x,x}(a,\tau) \right\rangle \right\|^{1/2} \left\| \left\langle W_{y,y}(a,\tau) \right\rangle \right\|^{1/2}},$$

where values of $R_{x,y}(a,\tau)$ ranges from 0 (independent time series) to 1 (phase-locked time series). Brackets indicate a smoothing function over local variation in the frequency and temporal domains. , and without this smoothing over local variability in the spectra, $R_{x,y}(a,\tau) = 1$ for all a, τ. Details of this smoothing can alter the strongly influence $R_{x,y}(a,\tau)$, with smaller smoothing windows producing higher values and and larger smoothing windows showing more variability. However, the qualitative results, e.g., relative rank of coherence within the spectrum and significance testing, remain unchanged (Torrence and Webster 1999). By default, the Matlab package uses a boxcar window based on the dimensions of the spectrum: 1/20th the number of scales and 1/10th the number of time steps. In our simulation model the width in the time domain was therefore 100 time steps but only 4 time steps in the gypsy moth analyses. Given that the qualitative results remain identical, we changed the smoothing width to 10 time steps in gypsy moth analysis to improve readability of Figure 6. .

We tested for significant periodicity using a Hidden Markov Model simulation experiment, which accounts for short-term autocorrelation present in the time series, without making assumptions about the structure of that autocorrelation (Cazelles et al. 2014). Values of $x(t)$ are empirically binned into a histogram, and a transition matrix is fit among these bins. The transition matrix is used to generate 1000 surrogate time series to which wavelet transform is fit. Values in the empirical spectrum above the 95th percentile of the simulated values are considered statistically significant. Bivariate analyses were subject to an analogous test based on transition matrices of $x(t)$ and $y(t)$.

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