

## Appendix B. Additional methods: Calculation of heat fluxes.

As in Goudsmit et al. (2002) the heat budget considers heat fluxes due to short wave radiation  $H_S$ , long wave radiation from the atmosphere  $H_A$ , long wave radiation from the water surface  $H_W$ , evaporation  $H_E$  and conduction  $H_C$ . The net heat flux  $H$  is calculated as

$$H = H_S + H_A + H_W + H_E + H_C \quad (\text{B.1})$$

The heat fluxes are calculated from meteorological data and surface water temperatures obtained from the lake model using the bulk formulae of Livingstone and Imboden (1989) in case of  $H_W$ ,  $H_E$  and  $H_C$  (see Goudsmit et al. 2002) and Izomon et al. (2003) in case of  $H_A$ . Data on short wave radiation  $S$  were available from the meteorological station and  $H_S$  was calculated by estimating reflection of short wave radiation according to Ollinger (1999) assuming a smooth water surface.

### *Meteorological and lake data required for the calculation of the heat fluxes*

$T_a$ : air temperature [ $^{\circ}\text{C}$ ]

$T_w$ : surface water temperature [ $^{\circ}\text{C}$ ]

$h$ : relative humidity [-]

$S$ : solar radiation [ $\text{Wm}^{-2}$ ]

$Cl$ : cloud cover [-]

$p_{air}$ : air pressure [hPa]

$u_{10}$ : wind speed 10 m above ground [ $\text{m s}^{-1}$ ]

Vapor pressure of water in the atmosphere  $VP_a$  and immediately at the lake surface  $VP_w$  were calculated after Gill (1982):

$$VP_a = h \cdot (1.0+10^{-6} \cdot p_{air} \cdot (4.5+6 \cdot 10^{-4} \cdot T_a \cdot T_a)) \cdot 10^{(0.7859+0.03477 \cdot T_a)/(1+0.00412 \cdot T_a)} \quad [\text{hPa}]$$

(B.2)

$$VP_w = (1.0+10^{-6} \cdot p_{air} \cdot (4.5+6 \cdot 10^{-4} \cdot T_w \cdot T_w)) \cdot 10^{(0.7859+0.03477 \cdot T_w)/(1+0.00412 \cdot T_w)} \quad [\text{hPa}] \quad (\text{B.3})$$

***Bulk formulae for the heat fluxes***

All heat fluxes are defined positive for heat fluxes directed from the atmosphere to the lake.

*Heat flux due to long wave radiation from the atmosphere based on Iziomon et al. (2003)*

$$H_A = p_l \cdot (1-r_a) \cdot R_A \quad (\text{B.4})$$

$$R_A = 5.67e-8 \cdot (273.15+T_a)^4 (1+0.0035 \cdot Cl \cdot Cl \cdot 64) \cdot (1-0.35 \cdot e^{(-10 \cdot VP_a / (T_a+273.15))}) \quad (\text{B.5})$$

long wave radiation from the atmosphere (Iziomon et al. 2003)

$r_a = 0.03$  ratio of reflected long-wave irradiance

$p_l = 0.99527$  calibration parameter (see also Goudsmit et al. 2002)

*Heat flux due to short wave irradiance (Ollinger 1999)*

$$H_S = (1-r_{dir}) \cdot S_{dir} + (1-r_{diff}) \cdot S_{diff} \quad (\text{B.6})$$

$$S_{dir} = (0.8 - 0.8 Cl) \cdot S \quad \text{direct solar radiation}$$

(B.7)

$$S_{diff} = (0.2 + 0.8 Cl) \cdot S \quad \text{diffuse short wave radiation} \quad (\text{B.8})$$

$r_{dir}$ : fraction of reflected direct solar radiation, which is calculated from the Fresnel equations and an 1.33 as index

of refraction.  $r_{dir}$  depends on the angle of the incident light and therefore varies with time.

$$r_{diff} = (Cl \cdot 0.05 + (1 - Cl) \cdot r_{diff,c}) \quad \text{fraction of reflection diffusive} \quad (B.9)$$

short wave radiation

$r_{diff,c}$ : fraction of reflected diffusive short wave radiation for clear sky conditions based on Dirmhirn (1964).  $r_{diff}$  depends on the zenith angle and therefore varies with time.

*Heat flux due to long-wave radiation from the water surface (Livingstone and Imboden 1989)*

$$H_W = -0.97 \cdot 5.67 \cdot 10^{-8} \cdot (T_w + 273.15)^4 \quad (B.10)$$

*Heat flux due to evaporation (Latent heat flux) (Livingstone and Imboden 1989)*

$$H_E = -f_u^* \cdot (VP_W - VP_A) \quad (B.11)$$

$$f_u = 4.4 + 1.82 \cdot u_{10} + 0.26 \cdot (T_w - T_a) \quad (B.12)$$

$p_2 = 0.97521$  calibration parameter (see also Livingstone and Imboden 1989 and Goudsmit et al. 2002)

$$f_u^* = p_2 \cdot f_u \quad (B.13)$$

*Sensible heat flux (Livingstone and Imboden 1989)*

$$H_C = -f_u^* \cdot B_0 \cdot (T_w - T_a) \quad (B.14)$$

$B_0 = 0.61$  Bowen constant



LITERATURE CITED

- Drimhirm, I. 1964. Das Strahlungsfeld im Lebensraum. Akad. Verl.-G., Frankfurt am Main.  
426pp.
- Gill, A. E. 1982. Atmosphere-ocean dynamics. Academic Press.
- Goudsmit, G-H., H. Burchard, F. Peeters, and A. Wüest. 2002. Application of k- $\epsilon$  turbulence models to lakes - the role of internal seiches. *Journal of Geophysical Research* 107:3230–3242.
- Iziomon, M.G., Mayer H., Matzarakis A. 2003. Downward atmospheric longwave irradiance under clear and cloudy skies: Measurement and parameterization. *Journal of Atmospheric and Solar-Terrestrial Physics* 65:1107–1116.
- Livingstone, D. M., and D. M. Imboden. 1989. Annual heat balance and equilibrium temperature of Lake Aegeri, Switzerland. *Aquatic Sciences* 51:351–369.
- Ollinger, D. 1999. Modellierung von Temperatur, Turbulenz und Algenwachstum mit einem gekoppelten physikalisch-biologischen Modell. Dissertation, Ruprechts-Karls-Universität, Heidelberg, ISBN 3-933342-38-4, 200pp.