Appendix A - Effects of spatially variable LEP values.

If LEP values, e_j, are not uniform over space, we define the matrix $\mathbf{P}_\sigma = diag(e_j)$ and decompose it as $\mathbf{P}_\sigma = e\mathbf{I} + \sigma\mathbf{P}'$, where e and σ are the mean and standard deviation of LEP, $\mathbf{I}$ is the identity matrix and $\mathbf{P}'$ a diagonal matrix whose diagonal elements have zero mean and standard deviation equal to 1. We define $\mathbf{M}_\sigma = \mathbf{P}_\sigma\mathbf{C}$ so that Eq. 4 (see main text) now reads $\mathbf{N}_{t+1} = \mathbf{M}_\sigma\mathbf{N}_t$. We call λ_{M_σ} the dominant eigenvalue of $\mathbf{M}_\sigma$ and $\mathbf{N}_{\mathbf{M}_\sigma}$ the positive associated eigenvector of norm 1 (considering $\|.\|_1$). By continuity of eigenvalues, λ_{M_σ} and $\mathbf{N}_{\mathbf{M}_\sigma}$ tend respectively toward $e\lambda_C$ and $\mathbf{N}_\mathbf{C}$ (the non-negative eigenvector associated to the dominant eigenvalue λ_C of the connectivity matrix $\mathbf{C}$) when σ tends toward 0. Furthermore, when the matrix $\mathbf{C}$ is primitive and irreducible, we have the following asymptotic property (see proof in appendix B):

$$\begin{cases} \lambda_{M_\sigma} = e\lambda_C + O(\sigma) \\ \mathbf{N}_{\mathbf{M}_\sigma} = \mathbf{N}_\mathbf{C} + O(\sigma) \end{cases}. \tag{A.1}$$

Eq. 4 reads $\mathbf{N}_{t+1} = (e\mathbf{I} + \sigma\mathbf{P}')\mathbf{C}\mathbf{N}_t$. Asymptotically we have $\mathbf{N}_{t+1} = \lambda_{M_\sigma}\mathbf{N}_t = (e\lambda_C + O(\sigma))\mathbf{N}_t$. From these two equalities, and because $\sigma\mathbf{P}'\mathbf{C}/e = O(\sigma)$, we conclude that

$\mathbf{C}\mathbf{N}_{t+1} = \lambda_C\mathbf{N}_t + O(\sigma)\mathbf{N}_t$, i.e., $\sum_{j=1}^{n} c_{ij}N_{j,t} = \lambda_C N_{i,t} + O(\sigma)N_{i,t}$ for all i. Therefore $LR_i = \lambda_C SR_i(\sigma) + O(\sigma)$ and from equation (C1) we obtain $SR_i(\sigma) = SR_i + O(\sigma)$ for all i so that, finally, $LR = \lambda_C SR + O(\sigma)$, asymptotically.

Combining the asymptotic relationships $\lambda_{M_\sigma} = e\lambda_C + O(\sigma)$ and $LR = \lambda_C SR + O(\sigma)$ on each patch, we obtain $\lambda_{M_\sigma} = e \times LR / SR + O(\sigma)$ where e and σ are the mean and standard deviation of LEP.