

Simulation descriptions and results

To explore the effects of SCR on inference for a spatial predictor of interest, we simulated 16 sets of 1000 data-sets each. The 16 data-sets represent all possible combinations of 2 spatial predictor types, 4 autocorrelation types, and 2 effect sizes for the spatial predictor.

Each data-set was constructed of the following components:

- $N = 30$ observations equally spaced along a one-dimensional transect, so the effective geographic locations G_i were $G_1 = 1, G_2 = 2, \dots, G_{30} = 30$.
- Two predictors:
 - A predictor X_i with a spatial relationship (spatial predictor, with one of two forms described below).
 - A predictor W_i with no spatial relationship to serve as comparison and check on computations.

Here we report results for only X , as the Type I error probabilities for a non-spatial predictor are uninfluenced by spatial autocorrelation.

- Errors ε_i : Errors were simulated according to one of four autocorrelation mechanisms, described below.
- Response: The response values were obtained as

$$Y_i = 1 + \beta_1 X_i + \beta_2 W_i + \varepsilon_i$$

where we considered two possible true values of the coefficient β_1 : 0 or 1.

Predictor Options:

(P1) Spatial Trend Predictor

For this option, predictor X_i has a mean trend determined by the spatial location. In particular, we used the following definition for X_i :

$$X_i = \frac{1}{3}G_i + \tau_i$$

where τ_i were independent $N(0, 2)$.

(P2) Spatially Autocorrelated Predictor

For this option, the mean of predictor X_i does not depend on the spatial location, but the values of X_i and X_j will be close if locations i and j are close. In particular, we used the following definition for X_i :

$$(X_1, X_2, \dots, X_{30})^T \sim MVN(\mathbf{0}_{n \times 1}, \mathbf{\Sigma})$$

where the $\{i, j\}$ element of the covariance matrix $\mathbf{\Sigma}$ is given by

$$\sigma_{i,j} = 0.75^{|i-j|},$$

that is, the correlation between the X_i is AR(1) with $\rho = 0.75$.

Autocorrelation Options:

(A0) No Autocorrelation

Errors ε_i were generated according to

$$\varepsilon_i \sim_{iid} N(0, 1)$$

(A1) Missing Spatial Predictor

Errors included the effect of a spatial trend predictor that was not measured or included in the model. In particular, the true model was given by

$$Y_i = 1 + \beta_1 X_i + \beta_2 W_i + \beta_3 Z_i + \varepsilon_i$$

where Z_i is a spatial predictor that is not included in the model fit, with form given by

$$Z_i = \frac{1}{6} G_i + \xi_i$$

where ξ_i were independent $N(0, 2)$, β_3 was set to 1, and ε_i were independent $N(0, 1)$.

(A2) Autocorrelated Errors

Errors were distributed with spatial autocorrelation, so errors corresponding to locations that were close together had similar values. In particular, we used the following definition for ε_i :

$$(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_{50})^T \sim MVN(\mathbf{0}_{n \times 1}, \mathbf{\Sigma})$$

where the $\{i, j\}$ element of the covariance matrix $\mathbf{\Sigma}$ is given by

$$\sigma_{i,j} = 0.75^{|i-j|},$$

that is, the correlation between the ε_i is AR(1) with $\rho = 0.75$.

(A3) Location Effect

Errors included a direct spatial contribution. In particular, the errors ε_i were defined as

$$\varepsilon_i = \frac{1}{100} (G_i - 10)^2 + \eta_i$$

where η_i were independent $N(0, 1)$.

We recorded the following results for each simulated data set:

1. Coefficient estimate for predictor of interest: $\hat{\beta}_1$
2. Standard error estimate for $\hat{\beta}_1$: $s_{\hat{\beta}_1}$
3. p -value for testing $H_0 : \beta_1 = 0$
4. 95% Confidence interval for β_1 :

$$CI = \left(\hat{\beta}_1 - t_{\alpha/2} s_{\hat{\beta}_1}, \hat{\beta}_1 + t_{\alpha/2} s_{\hat{\beta}_1} \right)$$

where the critical value $t_{\alpha/2}$ is the upper $\alpha/2$ quantile of a t -distribution with the appropriate degrees of freedom df .

To assess the performance of the methods considered, we calculate the following metrics across all 1000 simulations for each given scenario:

1. Average of the estimated coefficients $\hat{\beta}_1$
2. Standard error of the estimated coefficients $SE(\hat{\beta}_1)$
3. Rejection rate: What proportion of the 1000 simulated data-sets produced a p -value for testing $H_0 : \beta_1 = 0$ that was less than the desired significance level $\alpha = 0.05$? If the inference for a method is accurate, we expect this proportion to be near $0.05 = 5\%$ when the null hypothesis is true ($\beta_1 = 0$), and we want this proportion to be as high as possible when the null hypothesis is false ($\beta_1 = 1$).

4. Confidence Interval coverage: What proportion of the 1000 simulated data-sets produced a 95% confidence intervals for β_1 that contained the true value of β_1 ? If the inference for a method is accurate, we would like the confidence interval coverage to be $0.95 = 95\%$.
5. The number of significant spatial components that are identified, for each of the three different weight matrices. We denote these numbers by n_{K1}, n_{K2} , and n_{K3} for the number of components identified using weight matrix W1, W2, or W3 respectively. These numbers are tabled for the 1000 simulated data-sets.

The tables and plots below are organized by simulation scenario (Predictor type, Autocorrelation type, and True Effect type).

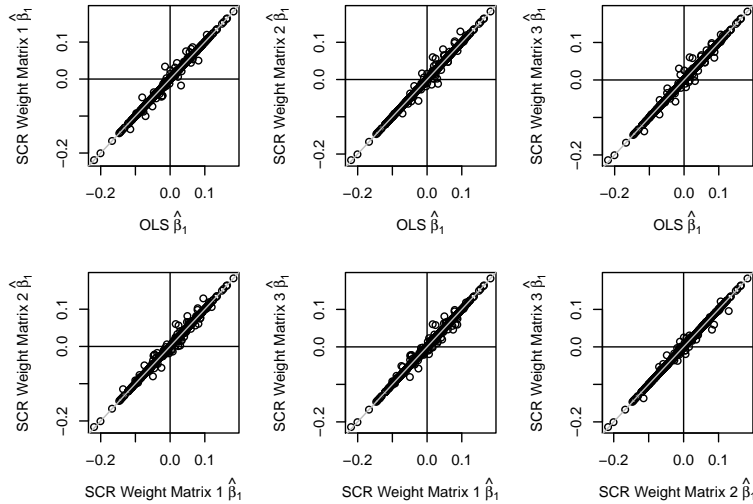
Simulation Results

Predictor: (P1) Spatial Trend Predictor
Autocorrelation: (A0) No autocorrelation
True Effect: (Null) $\beta_1 = 0$

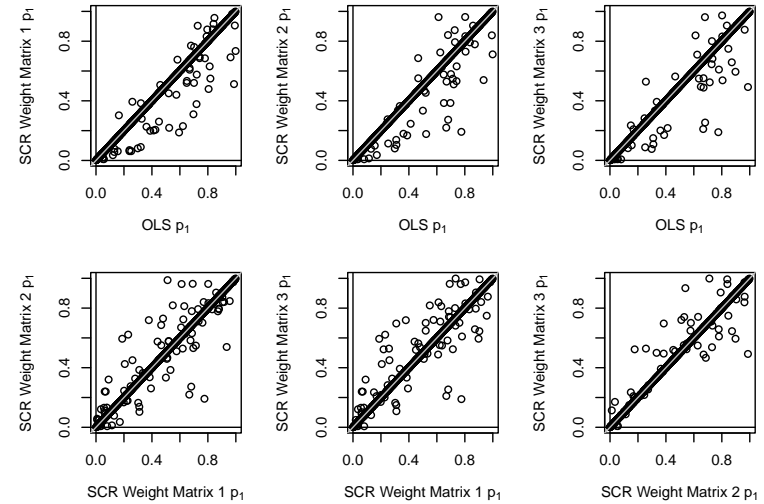
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	0	0.05	0.95
OLS	-0.001	0.053	0.947
SCR W1	-0.001	0.055	0.945
SCR W2	-0.001	0.056	0.944
SCR W3	-0.001	0.055	0.945
OLS + Loc	0.001	0.049	0.951

	0	1
n_{K1}	944	56
n_{K2}	949	51
n_{K3}	953	47

Comparison of Coefficient Estimates



Comparison of p-values

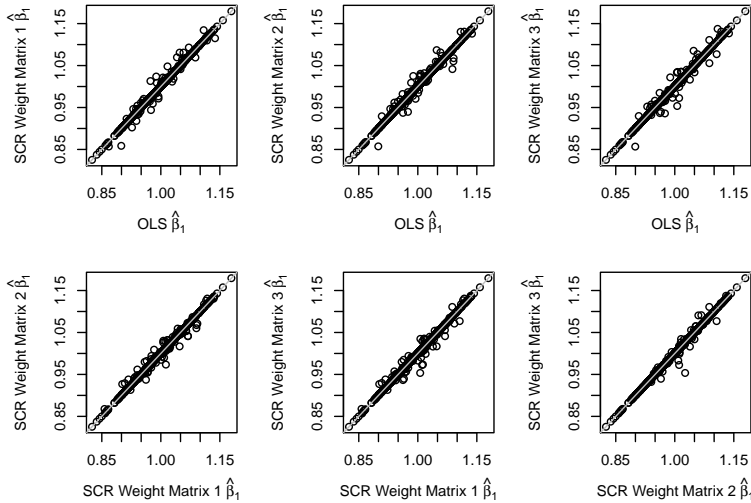


Predictor: (P1) Spatial Trend Predictor
 Autocorrelation: (A0) No autocorrelation
 True Effect: (Alternative) $\beta_1 = 1$

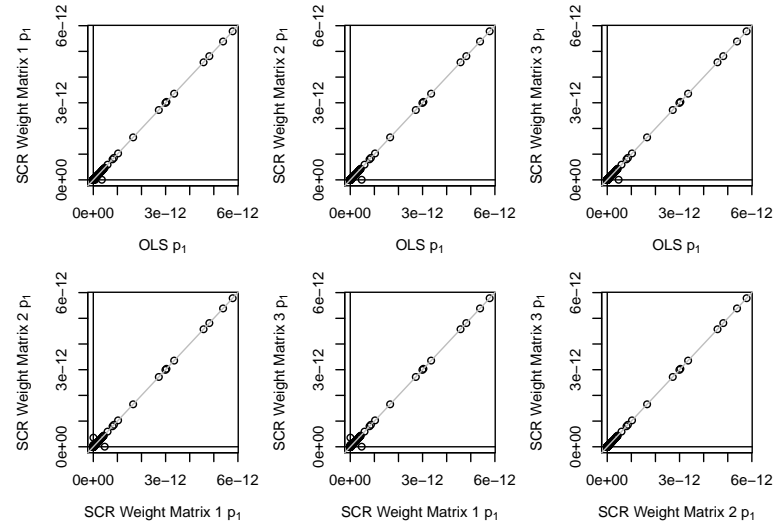
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	1	Large	0.95
OLS	1.000	1.000	0.953
SCR W1	1.000	1.000	0.949
SCR W2	1.000	1.000	0.951
SCR W3	1.000	1.000	0.949
OLS + Loc	0.995	1.000	0.945

	0	1
n_{K1}	944	56
n_{K2}	939	61
n_{K3}	940	60

Comparison of Coefficient Estimates



Comparison of p-values

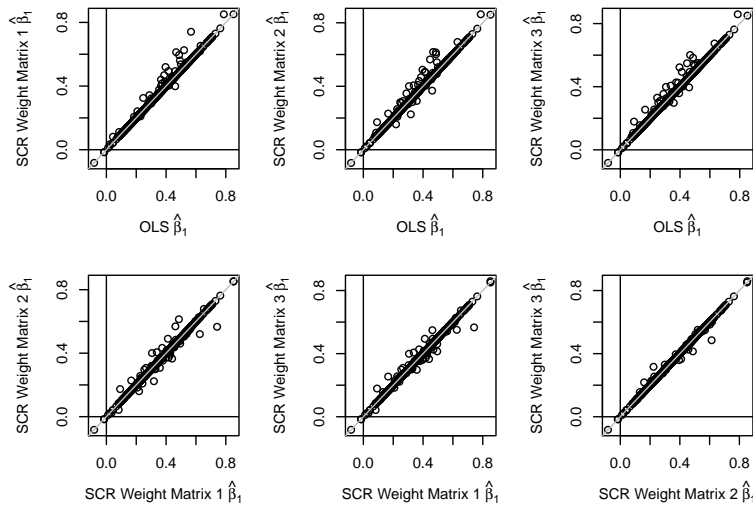


Predictor: (P1) Spatial Trend Predictor
Autocorrelation: (A1) Missing Spatial Predictor
True Effect: (Null) $\beta_1 = 0$

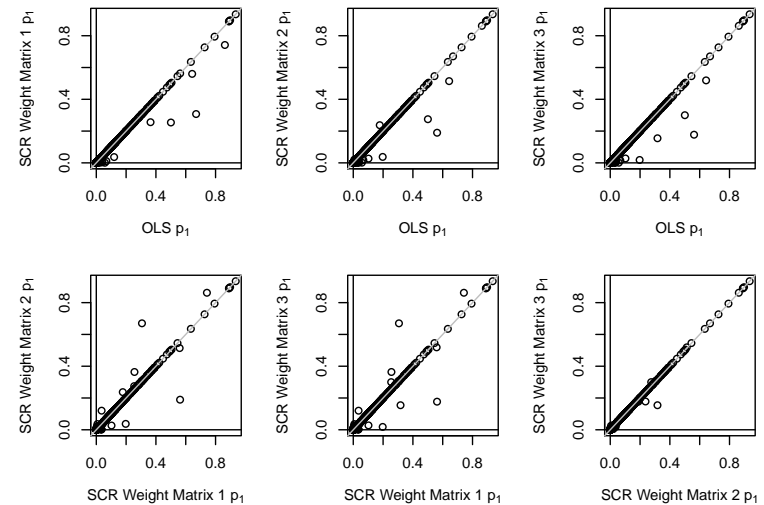
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	0	0.05	0.95
OLS	0.352	0.750	0.250
SCR W1	0.354	0.753	0.247
SCR W2	0.353	0.754	0.246
SCR W3	0.353	0.754	0.246
OLS + Loc	0.008	0.056	0.944

	0	1
n_{K1}	957	43
n_{K2}	954	46
n_{K3}	959	41

Comparison of Coefficient Estimates



Comparison of p-values

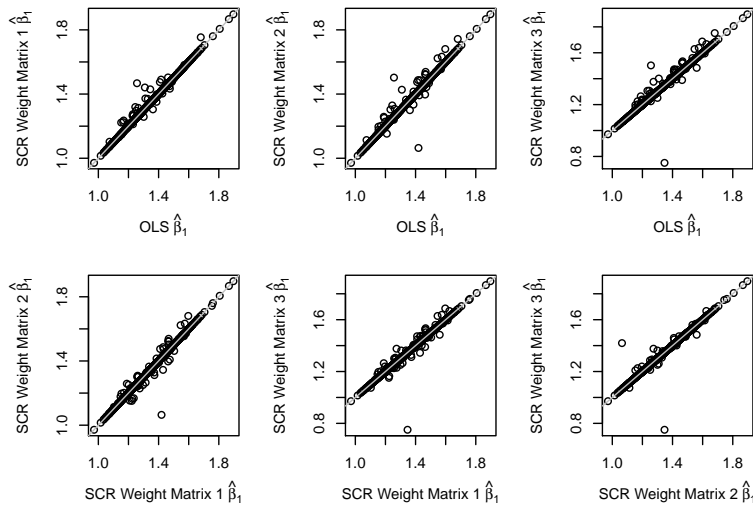


Predictor: (P1) Spatial Trend Predictor
Autocorrelation: (A1) Missing Spatial Predictor
True Effect: (Alternative) $\beta_1 = 1$

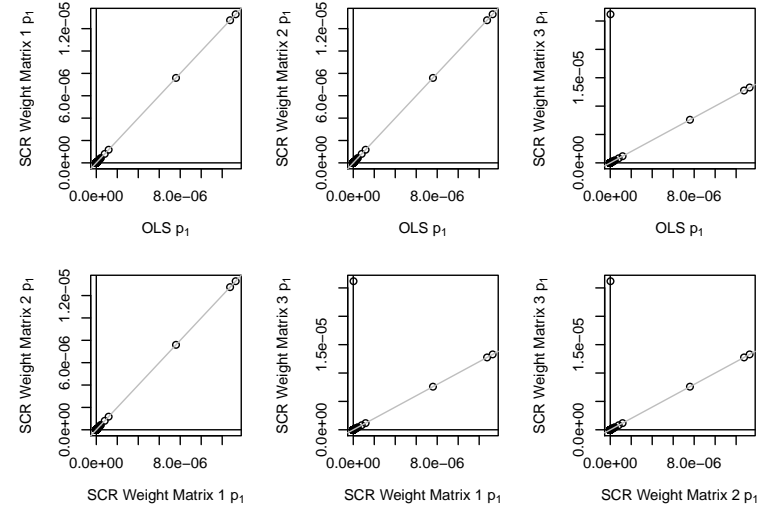
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	1	Large	0.95
OLS	1.348	1.000	0.260
SCR W1	1.349	1.000	0.252
SCR W2	1.349	1.000	0.252
SCR W3	1.349	1.000	0.250
OLS + Loc	0.995	0.983	0.941

	0	1
n_{K1}	963	37
n_{K2}	961	39
n_{K3}	953	47

Comparison of Coefficient Estimates



Comparison of p-values

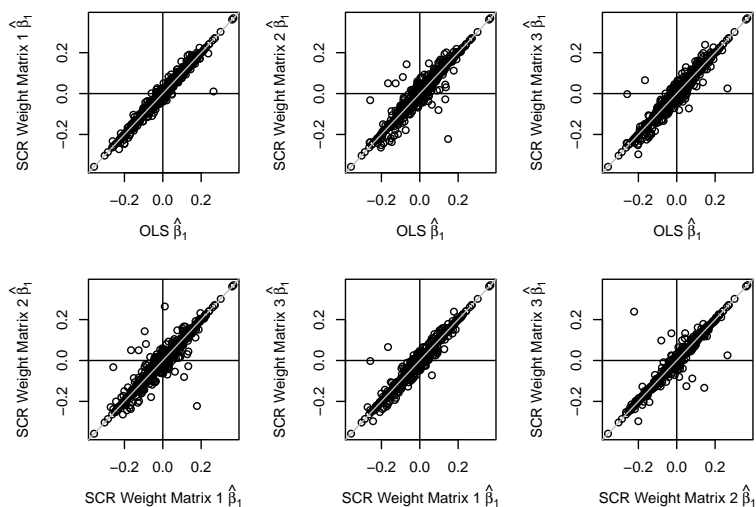


Predictor: (P1) Spatial Trend Predictor
 Autocorrelation: (A2) Autocorrelated Errors
 True Effect: (Null) $\beta_1 = 0$

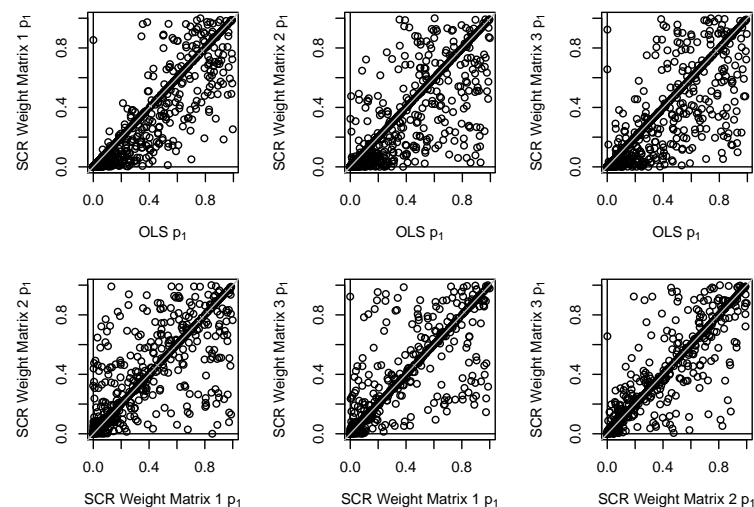
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	0	0.05	0.95
OLS	-0.001	0.326	0.674
SCR W1	-0.001	0.415	0.585
SCR W2	-0.002	0.398	0.602
SCR W3	-0.002	0.415	0.585
OLS + Loc	-0.003	0.043	0.956

	0	1	2
n_{K1}	499	483	18
n_{K2}	524	462	14
n_{K3}	512	475	13

Comparison of Coefficient Estimates



Comparison of p-values

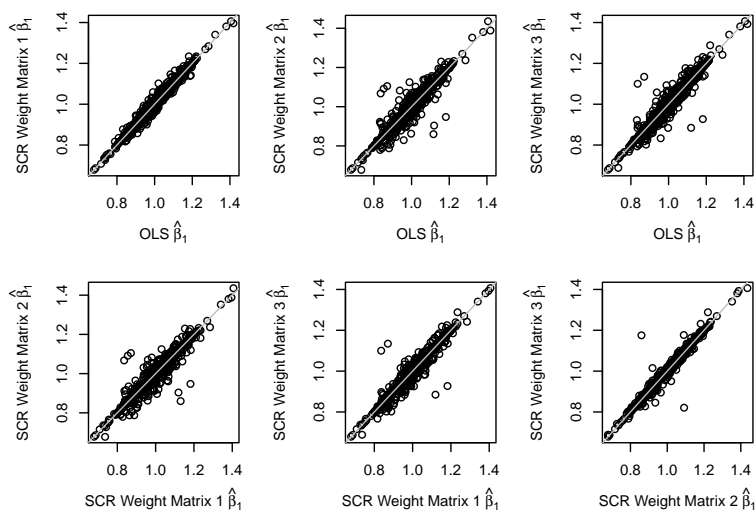


Predictor: (P1) Spatial Trend Predictor
Autocorrelation: (A2) Autocorrelated Errors
True Effect: (Alternative) $\beta_1 = 1$

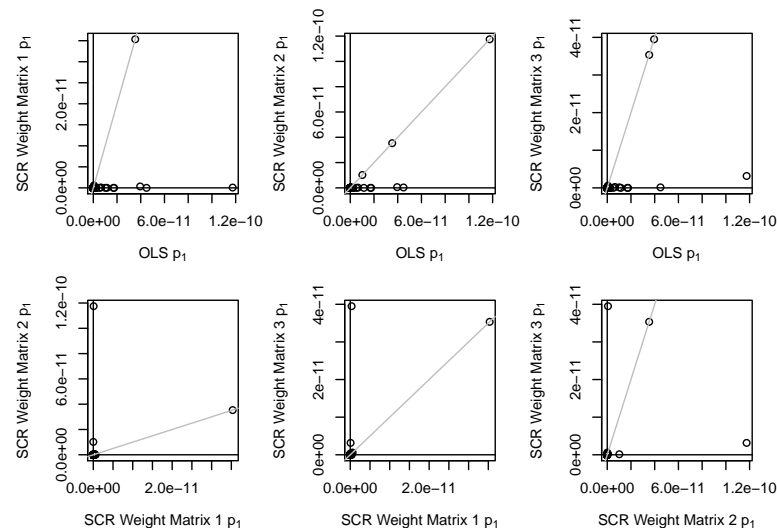
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	1	Large	0.95
OLS	1.002	1.000	0.641
SCR W1	1.002	1.000	0.552
SCR W2	1.001	1.000	0.561
SCR W3	1.002	1.000	0.546
OLS + Loc	1.000	1.000	0.940

	0	1	2
n_{K1}	513	474	13
n_{K2}	539	447	14
n_{K3}	532	457	11

Comparison of Coefficient Estimates



Comparison of p-values

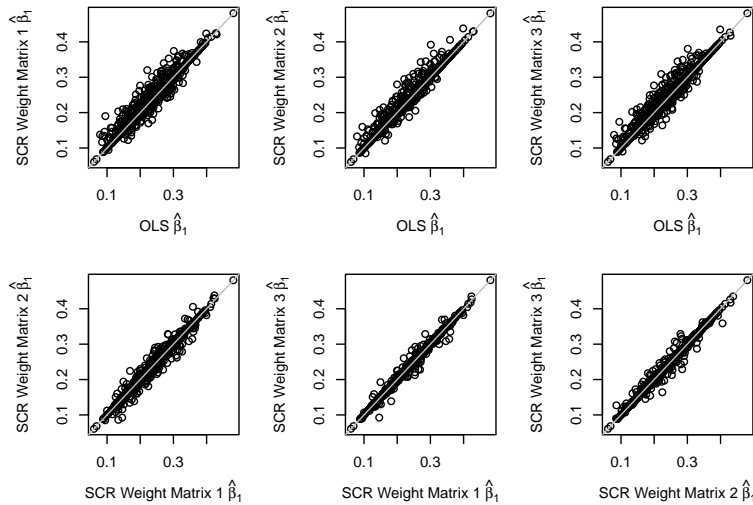


Predictor: (P1) Spatial Trend Predictor
Autocorrelation: (A3) Location Effect
True Effect: (Null) $\beta_1 = 0$

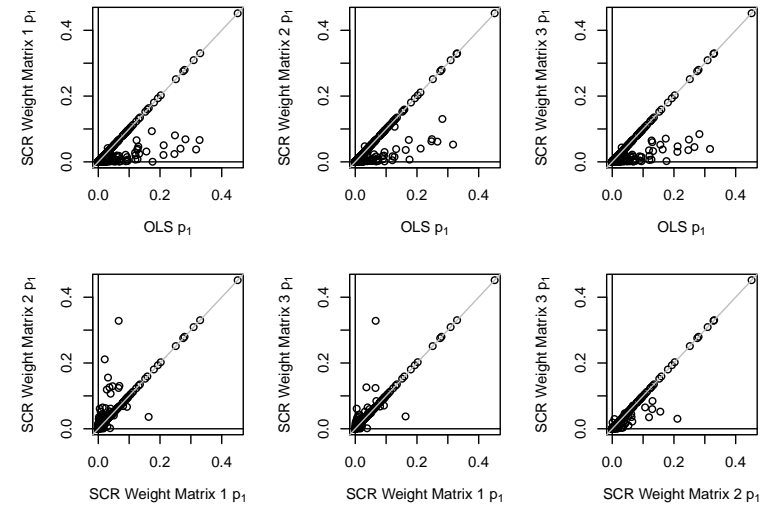
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	0	0.05	0.95
OLS	0.234	0.909	0.091
SCR W1	0.238	0.941	0.059
SCR W2	0.238	0.931	0.069
SCR W3	0.238	0.938	0.062
OLS + Loc	0.011	0.048	0.952

	0	1
n_{K1}	653	347
n_{K2}	760	240
n_{K3}	686	314

Comparison of Coefficient Estimates



Comparison of p-values

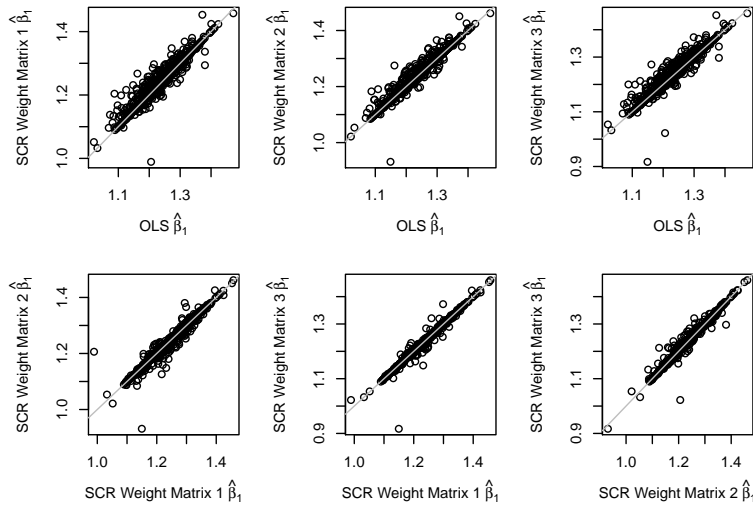


Predictor: (P1) Spatial Trend Predictor
 Autocorrelation: (A3) Location Effect
 True Effect: (Alternative) $\beta_1 = 1$

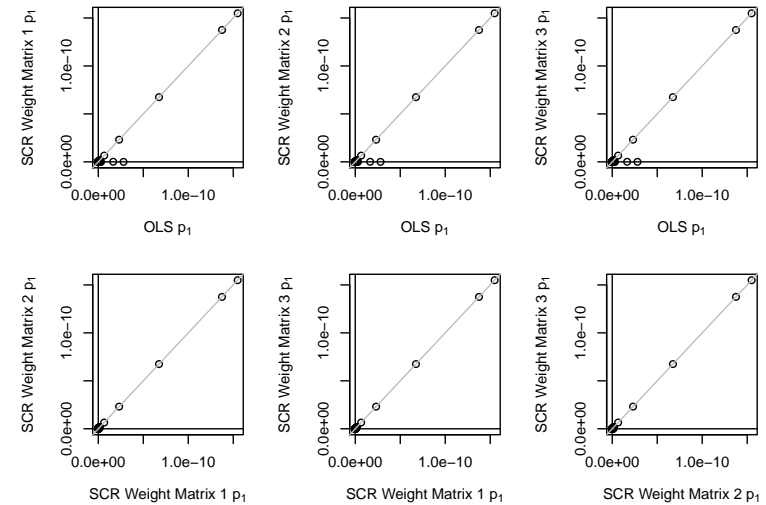
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	1	Large	0.95
OLS	1.231	1.000	0.080
SCR W1	1.235	1.000	0.054
SCR W2	1.234	1.000	0.064
SCR W3	1.234	1.000	0.056
OLS + Loc	1.003	1.000	0.944

	0	1	2
n_{K1}	650	348	2
n_{K2}	766	234	0
n_{K3}	680	319	1

Comparison of Coefficient Estimates



Comparison of p-values

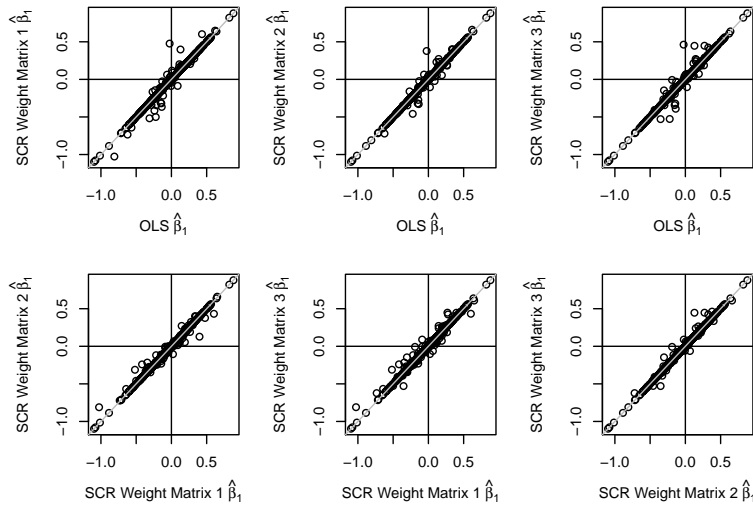


Predictor: (P2) Spatially Autocorrelated Predictor
 Autocorrelation: (A0) No autocorrelation
 True Effect: (Null) $\beta_1 = 0$

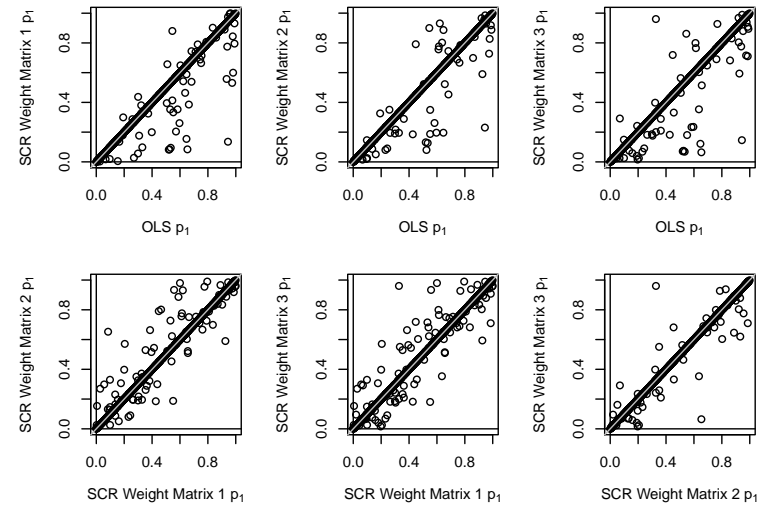
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	0	0.05	0.95
OLS	-0.019	0.043	0.957
SCR W1	-0.019	0.047	0.953
SCR W2	-0.019	0.046	0.954
SCR W3	-0.018	0.048	0.952
OLS + Loc	-0.022	0.045	0.955

	0	1
n_{K1}	940	60
n_{K2}	949	51
n_{K3}	941	59

Comparison of Coefficient Estimates



Comparison of p-values

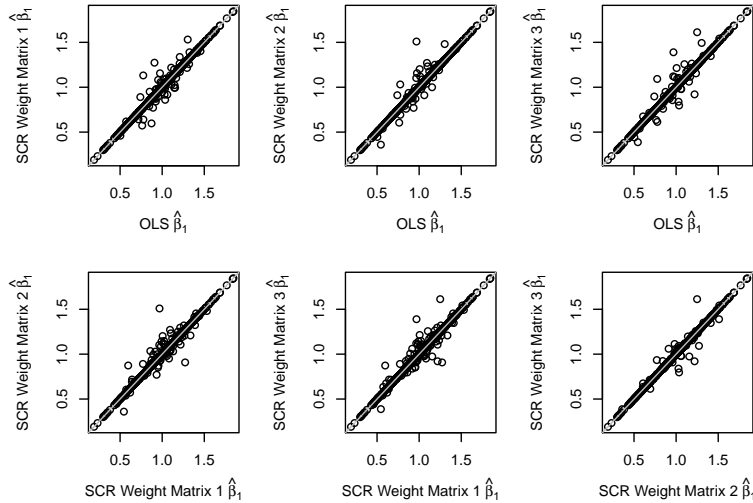


Predictor: (P2) Spatially Autocorrelated Predictor
 Autocorrelation: (A0) No autocorrelation
 True Effect: (Alternative) $\beta_1 = 1$

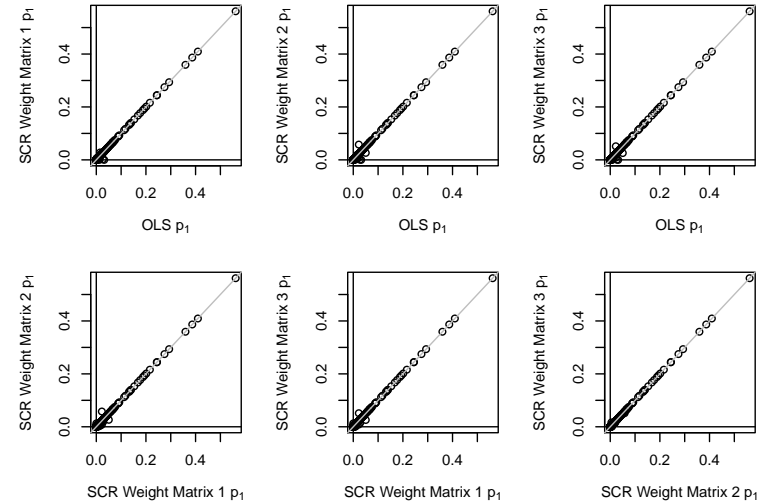
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	1	Large	0.95
OLS	0.998	0.962	0.944
SCR W1	0.998	0.962	0.940
SCR W2	0.999	0.962	0.939
SCR W3	0.999	0.962	0.939
OLS + Loc	0.998	0.936	0.945

	0	1	2
n_{K1}	937	62	1
n_{K2}	950	49	1
n_{K3}	946	53	1

Comparison of Coefficient Estimates



Comparison of p-values

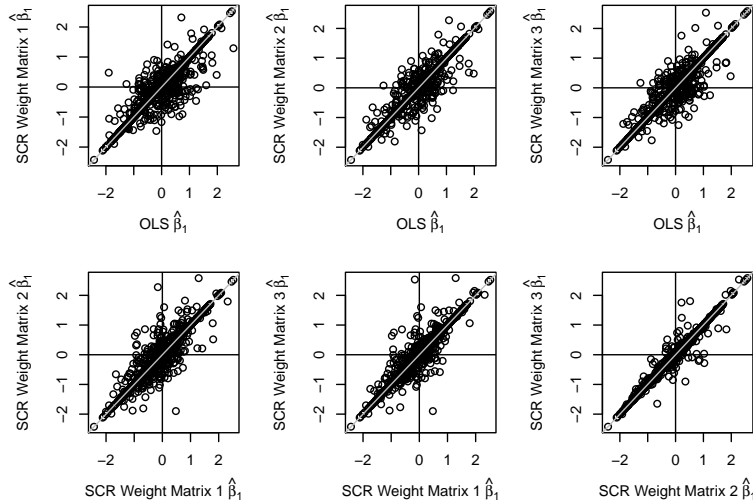


Predictor: (P2) Spatially Autocorrelated Predictor
Autocorrelation: (A1) Missing Spatial Predictor
True Effect: (Null) $\beta_1 = 0$

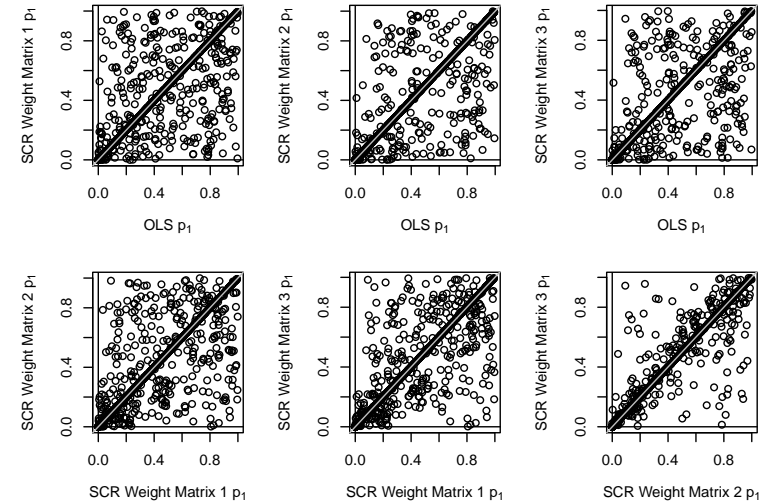
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	0	0.05	0.95
OLS	-0.030	0.200	0.800
SCR W1	-0.055	0.210	0.790
SCR W2	-0.043	0.214	0.786
SCR W3	-0.045	0.209	0.791
OLS + Loc	-0.052	0.053	0.947

	0	1	2
n_{K1}	659	341	0
n_{K2}	747	253	0
n_{K3}	714	285	1

Comparison of Coefficient Estimates



Comparison of p-values

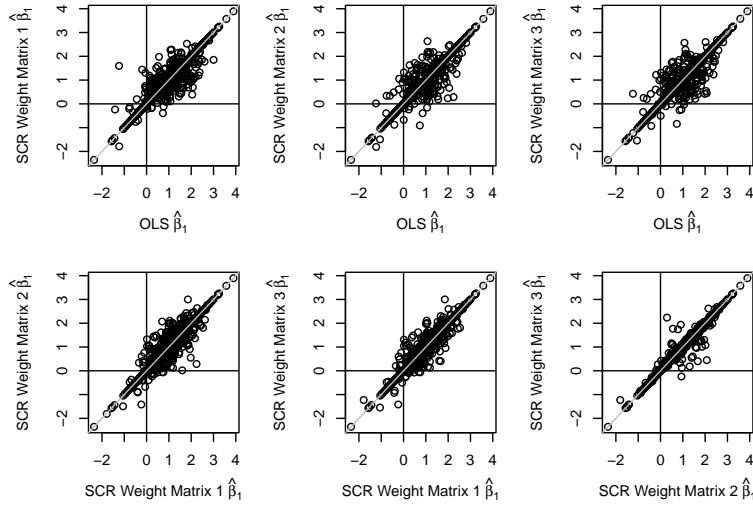


Predictor: (P2) Spatially Autocorrelated Predictor
Autocorrelation: (A1) Missing Spatial Predictor
True Effect: (Alternative) $\beta_1 = 1$

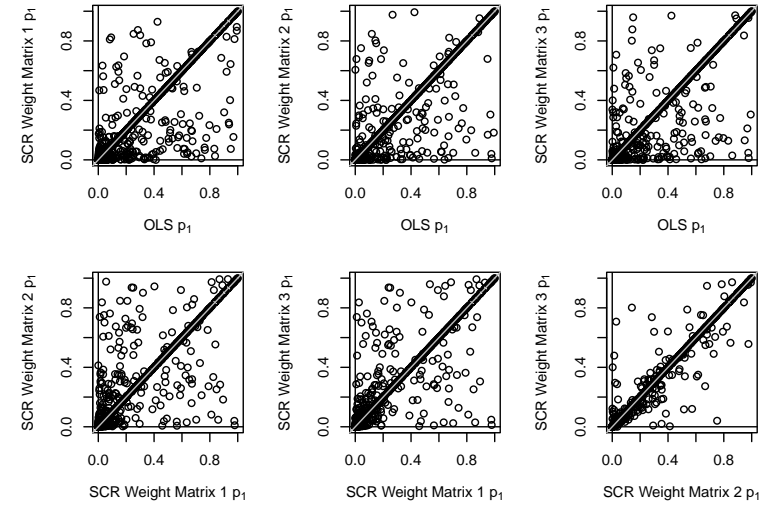
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	1	Large	0.95
OLS	1.001	0.421	0.799
SCR W1	0.990	0.460	0.788
SCR W2	0.997	0.462	0.786
SCR W3	0.993	0.458	0.790
OLS + Loc	1.009	0.441	0.955

	0	1
n_{K1}	684	316
n_{K2}	759	241
n_{K3}	730	270

Comparison of Coefficient Estimates



Comparison of p-values

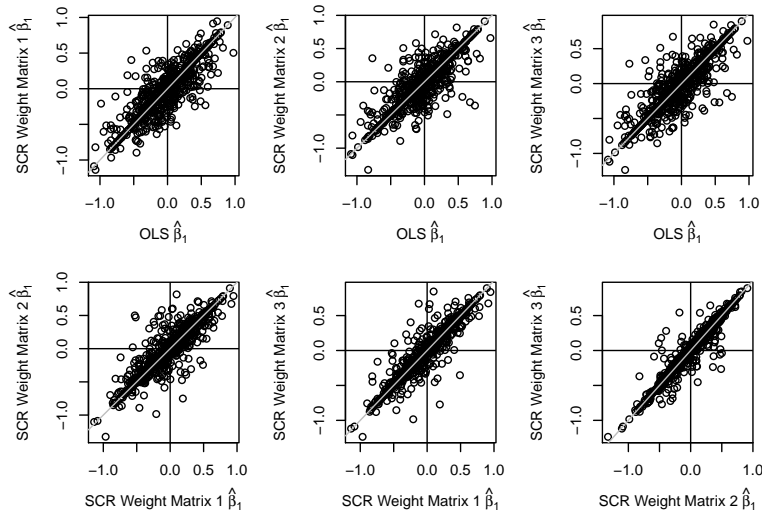


Predictor: (P2) Spatially Autocorrelated Predictor
Autocorrelation: (A2) Autocorrelated Errors
True Effect: (Null) $\beta_1 = 0$

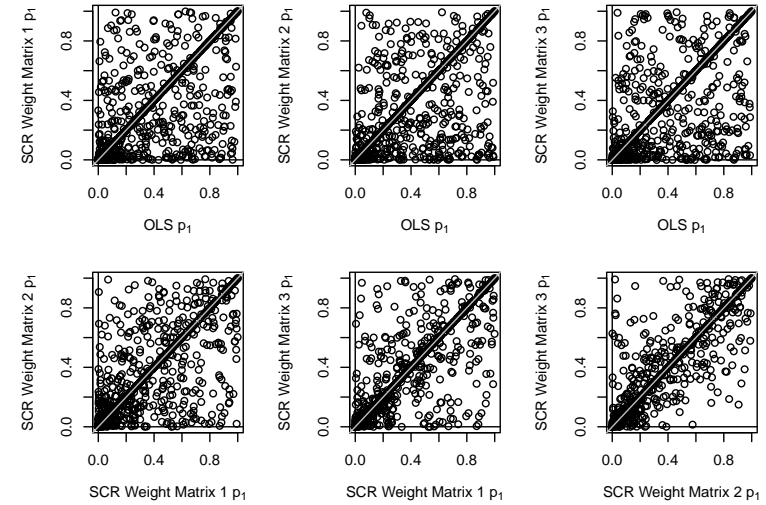
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	0	0.05	0.95
OLS	-0.009	0.248	0.752
SCR W1	-0.012	0.308	0.692
SCR W2	-0.009	0.319	0.681
SCR W3	-0.012	0.324	0.676
OLS + Loc	-0.013	0.237	0.760

	0	1	2
n_{K1}	506	481	13
n_{K2}	504	486	10
n_{K3}	514	477	9

Comparison of Coefficient Estimates



Comparison of p-values

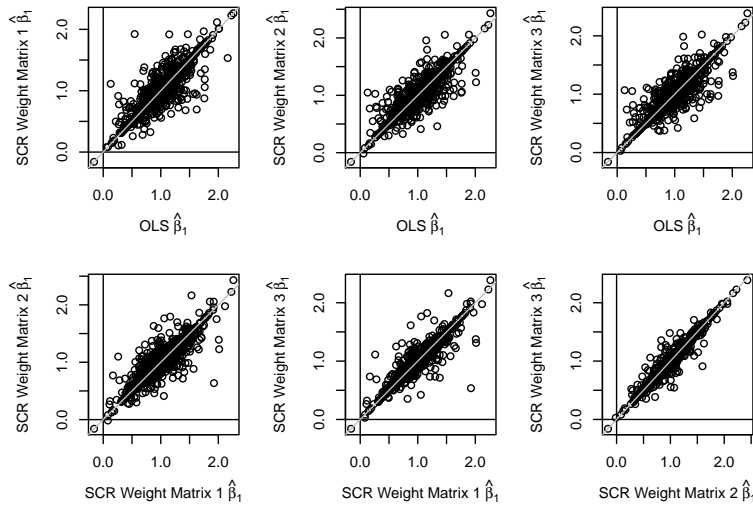


Predictor: (P2) Spatially Autocorrelated Predictor
Autocorrelation: (A2) Autocorrelated Errors
True Effect: (Alternative) $\beta_1 = 1$

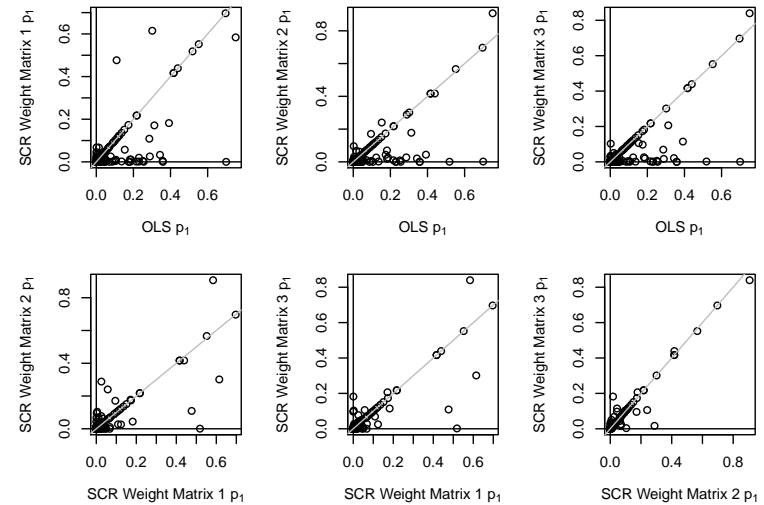
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	1	Large	0.95
OLS	1.017	0.949	0.774
SCR W1	1.019	0.969	0.694
SCR W2	1.018	0.968	0.719
SCR W3	1.019	0.968	0.719
OLS + Loc	1.009	0.944	0.797

	0	1	2
n_{K1}	521	463	16
n_{K2}	520	467	13
n_{K3}	513	475	12

Comparison of Coefficient Estimates



Comparison of p-values

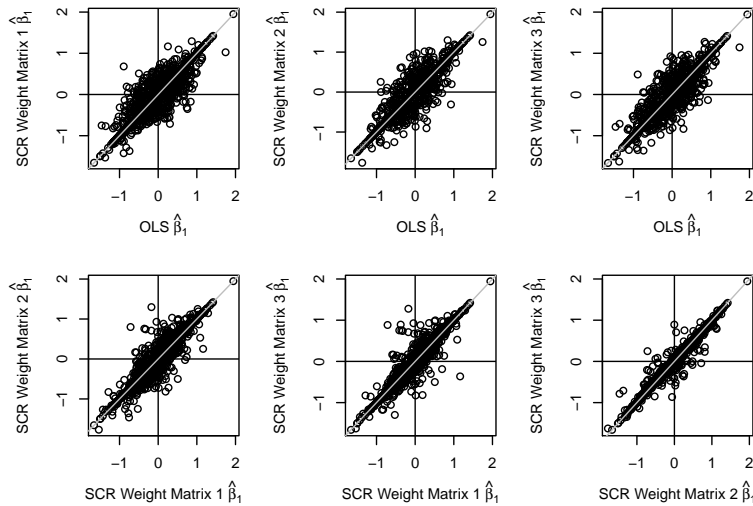


Predictor: (P2) Spatially Autocorrelated Predictor
Autocorrelation: (A3) Location Effect
True Effect: (Null) $\beta_1 = 0$

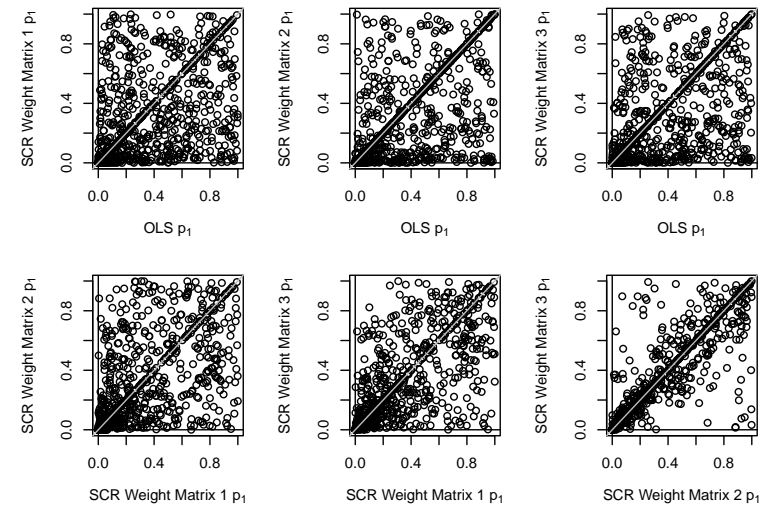
	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	0	0.05	0.95
OLS	0.013	0.306	0.694
SCR W1	0.016	0.377	0.623
SCR W2	0.017	0.397	0.603
SCR W3	0.017	0.398	0.602
OLS + Loc	0.011	0.175	0.823

	0	1	2
n_{K1}	365	607	28
n_{K2}	513	480	7
n_{K3}	430	548	22

Comparison of Coefficient Estimates



Comparison of p-values

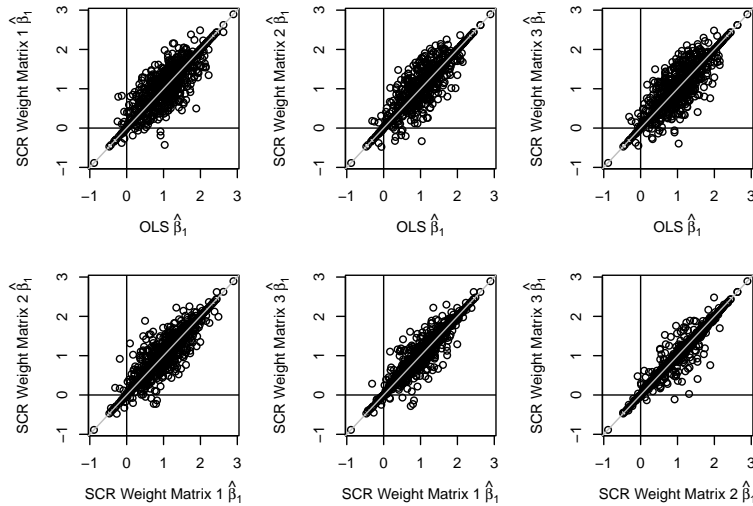


Predictor: (P2) Spatially Autocorrelated Predictor
Autocorrelation: (A3) Location Effect
True Effect: (Alternative) $\beta_1 = 1$

	Coef. Est. Mean	Rejection Rate	CI Coverage
Target Values	1	Large	0.95
OLS	1.016	0.679	0.689
SCR W1	1.023	0.738	0.603
SCR W2	1.033	0.721	0.605
SCR W3	1.030	0.729	0.592
OLS + Loc	1.022	0.814	0.820

	0	1	2
n_{K1}	397	579	24
n_{K2}	545	450	5
n_{K3}	447	541	12

Comparison of Coefficient Estimates



Comparison of p-values

