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C Bias and correction of the FAP estimator

We tested the bias of the OUSS FAP estimator by numerically calculating the type-I error rate, i.e. the rate at which OUSS time series are erroneously rejected and classified as cyclic. We generated a large number of time series of the non-cyclic OUSS model described in the main article, while choosing σ, ρ (where $\rho = \exp(-\lambda\delta)$ is the correlation between subsequent time points) and ε/σ within a wide range. More precisely, σ, ρ and ε/σ were uniformly and randomly picked in the range $[0.01, 0.2]$, $(0, 1)$ and $[0, 2]$, respectively. We considered time series of varying length up to 4000. For each time series length (TSL), we ran $25\,000$ Monte Carlo simulations (only 5000 for TSL$= 4000$). Details on periodogram calculation and FAP estimation are given in B.

The FAP estimator is itself a random variable, whose cumulative distribution function (CDF) under the OUSS null hypothesis should ideally (i.e., in the absence of bias) be the identity function, $F(t) = t$. For example, at a 0.05 significance threshold, the null hypothesis should be rejected 5% of the times. Fig. 1 shows the obtained cumulative distribution functions for FAPs estimated using different TSLs. As can be seen, a considerable bias exists for low quality time series. For example, at a significance threshold 0.05 the type-I error rate is significantly lower than 5%. The bias disappears for longer time series. This observation comes to no surprise, as our FAP estimator assumes periodogram powers to be independent and exponentially distributed, which is only asymptotically true for long time series.

To account for this bias, the obtained CDF was applied to the estimated FAPs in all subsequent calculations. We remind the reader that for any continuous random variable X with CDF F, the random variable $F(X)$ is distributed within $[0, 1]$ and has CDF $G(t) = t$ (*diagonal*). Hence, by mapping the estimated FAPs to the corresponding CDF value, one obtains new rescaled estimators with a diagonal CDF. These rescaled estimators are, strictly speaking, not estimators for the tail of the original test statistic introduced in the main article. But since the applied CDF is monotonic, the rescaling preserves the order of the original FAPs. Hence, the rescaled (*corrected*) FAP remains suitable for testing against the OUSS process and exhibits a correct type-I error rate.

The CDF of the original FAPs is, strictly speaking, a conditional one, that is, it depends on the underlying OUSS process parameters as well as the sampling quality (TSL and time step). Nondimensionalization reduces the number of free parameters to 3, for example ρ (i.e. the correlation between subsequent time points), $\mathrm{NSR} = s_o/(\delta\varepsilon^2)$ (i.e. the power ratio of the

OU process to measurement error, or *noise to signal ratio*, at zero frequency) and TSL. We thus constructed a 4-dimensional rectangular grid spanning across different TSLs (ranging from 10 to 2000), estimated ρ values (ranging from 0 to 1), estimated NSR values (ranging from 0 to 5) and different FAP values (ranging from 0.0005 to 1). Each grid point was assigned the empirical CDF value at the particular FAP and for the particular TSL, after binning $400\,000$ Monte Carlo samples by their estimated ρ and NSR values. We used this grid to correct the FAPs estimated in all subsequent calculations. CDF values between grid points were approximated using multilinear interpolation (Andrews et al. 2013). Cases outside of the grid's domain were mapped to the closest grid value. This yielded a correct type-I error rate, e.g. a cycle detection rate of about 5% at a nominal significance level of 0.05. The precomputed grid and the appropriate FAP correction are included in the R package that we provide.

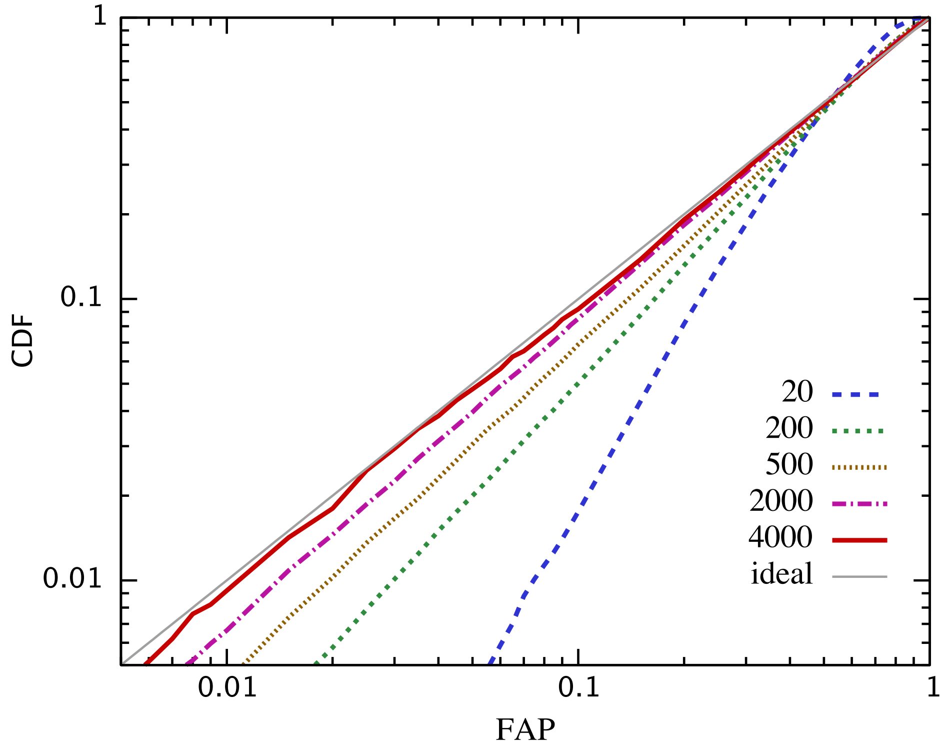

Fig. 1: Cumulative distribution function (CDF) of the uncorrected FAP estimator for randomly generated non-cyclic OUSS time series of different lengths. The grey diagonal corresponds to an ideal (i.e. unbiased) FAP estimator and is shown for comparison. The type-I error rate of an OUSS test at some significance threshold x is given by CDF(x). Technical details are given in C.

LITERATURE CITED

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