Appendix A of "Reproductive consequences of the timing of seasonal movements in a non-migratory wild bird population"

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Model description.— K equally spaced sampling occasions take place during the non-breeding season (NBS) and each bird in the population has its own capture-recapture-resight (CRR) history, of length $K+3$. The first K entries summarise the NBS with entries equal to 0, 1, 2 or 3 indicating whether the bird was, respectively, missed, caught, resighted or both at the corresponding sampling occasion. Entry $K+1$ is equal to 1 if the individual bird was detected incubating eggs (incubating stage) using the PIT-tag loggers and 0 otherwise, entry $K+2$ is equal to 1 if the individual was caught 7-11 days after its eggs had hatched (nestling stage), 2 if it was not caught but the nest was observed to be active and 0 otherwise, while entry $K+3$ is equal to 1 if the bird is known to have successfully fledged at least one chick (fledging stage) and 0 otherwise.

The time of first detection of birds with CRR history h is f_h and the time of last detection is l_h. For birds that were only detected in the breeding season (BS) $l_h \geq f_h > K$ and for birds that were detected at least once during the BS $l_h > K$ but f_h can be $\in \{1, \ldots, K\}$ or $> K$. The time of first capture during the NBS of birds with CRR history h is f_h^c with $f_h^c \in \{1, \ldots, K\}$ and f_h^c=NA for birds that were not caught during the NBS. Finally, the time of first resighting during the NBS is f_h^r with f_h^r=NA for birds that were not resighted during the NBS. The time of first possible detection is b, where $b \in \{1, \ldots, \min(f_h, K)\}$ since, all birds are assumed to have arrived before the end of the NBS. The time of last possible detection during the NBS is d, where $d \in \{\min(l_h, K), \ldots, K\}$. For birds detected during the BS $d = K$.

M stands for birds that were already marked at the start of the study and U for birds that were unmarked.

The number of M birds detected, either during the NBS or during the BS is denoted by D_M and the corresponding number of U birds by D_U.

The unique CRR histories observed for the D_M birds are summarised in matrix $\mathbf{X}_M$ with entry x_{hj}^M the jth entry of CRR history h, which is shared by n_h^M birds. These histories, including the history with all entries equal to 0 for the $N_M - D_M$ birds that were never detected, form the cells of a multinomial distribution with cell probabilities that are expressed as functions of the model parameters. The same holds for the unique CRR histories of the D_U birds which are summarised in matrix $\mathbf{X}_U$ and form the cells of a different multinomial distribution with n_h^U birds sharing the same history h. Although the $N = N_M + N_U$ birds are assumed to share all model parameters, we are required to model the two data sets using an integrated approach because the M birds, in contrast to the U birds, can be resighted without being caught and can be detected as soon as they start incubating eggs, something which is not possible for the U birds, unless they are caught during the NBS.

The probability of observing CRR history h for M birds is L_h^M and the probability of observing history ℓ for U birds is L_ℓ^U while the probabilities of not detecting one of the N_M and N_U birds are respectively L_0^M and L_0^U. Finally, the likelihood is the product of the two multinomial likelihoods that allocate the N_M and N_U to their corresponding unique CRR histories:

$$L = \frac{N_M!}{\prod_h n_h^M!(N_M - D_M)!} \prod_h (L_h^M)^{n_\ell^M} (L_0^M)^{N_M - D_M} \frac{N_U!}{\prod_\ell n_\ell^U!(N_U - D_U)!} \prod_\ell (L_\ell^U)^{n_\ell^U} (L_0^U)^{N_U - D_U} \tag{A.1}$$

The model parameters are given in Table A1.

TABLE A1: Definitions of model parameters. NBS stands for non-breeding season.

N	total number of birds that are present for at least one sampling occasion during the NBS
N_M	total number of marked birds (M) that are present for at least one sampling occasion during the NBS
N_U	total number of unmarked birds (U) that are present for at least one sampling occasion during the NBS
β_{j-1}	proportion of the N birds that are new additions to the population on sampling occasion j, with $\sum_{j=1}^{K} \beta_{j-1} = 1$
ϕ_j	probability that a bird present on occasion j is also present on occasion $j+1$
ψ_b	probability that a bird, that was a new arrival on occasion b, moves to the incubation stage, conditional on being present at the end of the NBS
ξ_b	probability that a bird, that was a new arrival on occasion b, moves from the incubation stage to the nestling stage
η_b	probability that a bird, that was a new arrival on occasion b, moves from the nestling stage to the fledging stage
p_j	probability that a bird present at the site on occasion j, that has not been caught during the course of this study, is captured
s_j	probability that a bird present at the site on occasion j, that has not used the feeders during the course of this study, is resighted
p'_j	probability that a bird present at the site on occasion j, that has been caught during the course of this study, is captured
s'_j	probability that a bird present at the site on occasion j, that has used the feeders during the course of this study, is resighted
s^B	probability of detecting a marked bird that is in the incubation stage
p^B	probability of capturing a bird that is in the nestling stage

The probability of observing any CRR history h for M birds that were detected, either during the NBS only and therefore have $l_h \leq K$ or during the BS, with or without being detected during the NBS, and therefore have $l_h > K$ and $d = K$ is:

$$
\begin{aligned}
L_h^M = \Bigg[& \sum_{b=1}^{f_h} \sum_{d=l_h}^{K} \beta_{b-1} \left\{ \prod_{j=b}^{d-1} \phi_j \right\} (1-\phi_d)^{\mathcal{I}(d<K)} \left\{ \prod_{j=b}^{d} (1-p_j) \right\}^{\mathcal{I}(f_h^c=\text{NA})} \\
& \times \left\{ \left(\prod_{j=b}^{f_h^c-1} (1-p_j) \right) p_{f_h^c} \left(\prod_{j=f_h^c+1}^{d} (p_j')^{\mathcal{I}(x_{hj}^M=1 \text{ or } 3)} (1-p_j')^{\mathcal{I}(x_{hj}^M=0 \text{ or } 2)} \right) \right\}^{\mathcal{I}(f_h^c \neq \text{NA})} \\
& \times \left\{ \prod_{j=b}^{d} (1-s_j) \right\}^{\mathcal{I}(f_h^r=\text{NA})} \\
& \times \left\{ \left(\prod_{j=b}^{f_h^r-1} (1-s_j) \right) s_{f_h^r} \left(\prod_{j=f_h^r+1}^{d} (s_j')^{\mathcal{I}(x_{hj}^M=2 \text{ or } 3)} (1-s_j')^{\mathcal{I}(x_{hj}^M=0 \text{ or } 1)} \right) \right\}^{\mathcal{I}(f_h^r \neq \text{NA})} \\
& \times \left\{ \psi_b(1-s_B)\xi_b(1-p_B) + \psi_b(1-s_B)(1-\xi_b) + 1 - \psi_b \right\}^{\mathcal{I}(d=K)} \Bigg]^{\mathcal{I}(l_h \leq K)} \\
& \times \Bigg[\sum_{b=1}^{\min(f_h,K)} \beta_{b-1} \left\{ \prod_{j=b}^{K-1} \phi_j \right\} \left\{ \prod_{j=b}^{K} (1-p_j) \right\}^{\mathcal{I}(f_h^c=\text{NA})} \\
& \times \left\{ \left(\prod_{j=b}^{f_h^c-1} (1-p_j) \right) p_{f_h^c} \left(\prod_{j=f_h^c+1}^{K} (p_j')^{\mathcal{I}(x_{hj}^M=1 \text{ or } 3)} (1-p_j')^{\mathcal{I}(x_{hj}^M=0 \text{ or } 2)} \right) \right\}^{\mathcal{I}(f_h^c \neq \text{NA})} \\
& \times \left\{ \prod_{j=b}^{K} (1-s_j) \right\}^{\mathcal{I}(f_h^r=\text{NA})} \\
& \times \left\{ \left(\prod_{j=b}^{f_r^c-1} (1-s_j) \right) s_{f_h^r} \left(\prod_{j=f_h^r+1}^{K} (s_j')^{\mathcal{I}(x_{hj}^M=2 \text{ or } 3)} (1-s_j')^{\mathcal{I}(x_{hj}^M=0 \text{ or } 1)} \right) \right\}^{\mathcal{I}(f_h^r \neq \text{NA})} \\
& \times \psi_b s_B^{\mathcal{I}(x_{h(K+1)}^M=1)} (1-s_B)^{\mathcal{I}(x_{h(K+1)}^M=0)} (\xi_b p_B)^{\mathcal{I}(x_{h(K+2)}^M=1)} \left\{\xi_b(1-p_B)\right\}^{\mathcal{I}(x_{h(K+2)}^M=2)} (1-\xi_b)^{\mathcal{I}(x_{h(K+2)}^M=0)} \\
& \times \eta_b^{\mathcal{I}(x_{h(K+3)}^M=1)} (1-\eta_b)^{\mathcal{I}(x_{h(K+3)}^M=0)} \Bigg]^{\mathcal{I}(l_h>K)} ,
\end{aligned}
\tag{A.2}
$$

where the indicator variable $\mathcal{I}(A)$ is equal to 1 if condition A is satisfied and 0 otherwise and all empty products are taken to be unity.

Details:

For M birds with $l_h \leq K$:

- $\sum_{b=1}^{f_h} \sum_{d=l_h}^{K} \beta_{b-1} \left\{ \prod_{j=b}^{d-1} \phi_j \right\} (1-\phi_d)^{\mathcal{I}(d<K)}$: models the presence history of the birds. These individuals arrived at an unknown time b which was sometime between the start of the study, 1, and their time of first detection, f_h, and either departed sometime after their last detection and before the end of the NBS or stayed until K.

- $\left\{ \prod_{j=b}^{d} (1-p_j) \right\}^{\mathcal{I}(f_h^c=\text{NA})}$: models the capture history of individuals that avoided capture throughout their duration of stay in the NBS and have f_h^c=NA.

- $\left\{ \left(\prod_{j=b}^{f_h^c-1} (1-p_j) \right) p_{f_h^c} \left(\prod_{j=f_h^c+1}^{d} (p_j')^{\mathcal{I}(x_{hj}^M=1 \text{ or } 3)} (1-p_j')^{\mathcal{I}(x_{hj}^M=0 \text{ or } 2)} \right) \right\}^{\mathcal{I}(f_h^c \neq \text{NA})}$: models the capture history of individuals that were caught at least once during the NBS and have $f_h^c \neq$NA. These individuals avoided capture from b until $f_h^c - 1$ and then were caught at f_h^c. Their capture history after their initial capture and until they departed is modelled in terms of p' since these individuals have been captured during the course of the study.

- $\left\{ \prod_{j=b}^{d} (1-s_j) \right\}^{\mathcal{I}(f_h^r=\text{NA})}$: models the resight history of individuals that avoided resighting throughout their duration of stay in the NBS and have f_h^r=NA.

- $\left\{ \left(\prod_{j=b}^{f_h^r-1} (1-s_j) \right) s_{f_h^r} \left(\prod_{j=f_h^r+1}^{d} (s_j')^{\mathcal{I}(x_{hj}^M=2 \text{ or } 3)} (1-s_j')^{\mathcal{I}(x_{hj}^M=0 \text{ or } 1)} \right) \right\}^{\mathcal{I}(f_h^r \neq \text{NA})}$: models the resight history of individuals that were resighted at least once during the NBS and have $f_h^r \neq$NA. These individuals avoided resighting from b until $f_h^r - 1$ and then were resighted at f_h^r. Their resight history after their initial resight and until they departed is modelled in terms of s' since these individuals have used the feeders during the course of the study.

- $\left\{\psi_b(1-s_B)\xi_b(1-p_B)+\psi_b(1-s_B)(1-\xi_b)+1-\psi_b\right\}^{\mathcal{I}(d=K)}$: models the breeding history. If these individuals remained until the end of the breeding season and therefore have $d=K$ then they either incubated eggs but were not detected using the scanner and were still present during the capture occasion in the BS but were not caught, or they incubated eggs but were not detected using the scanner and were not present during the capture occasion in the BS because they failed to hatch their eggs, or they did not breed at all.

For M birds with $l_h > K$ $d = K$ since they are known to have stayed until the BS and their CRR histories are similar to those for individuals with $l_h \leq K$ with the main difference being that d is known to be K. Also, they are known to have bred and have a breeding history which is modelled as shown in the last two rows of the expression for L_h^M.

The probability of the CRR history with all entries equal to 0 for M birds is:

$$\begin{aligned} L_0^M = &\sum_{b=1}^{K}\sum_{d=b}^{K}\beta_{b-1}\left\{\prod_{j=b}^{d-1}\phi_j\right\}(1-\phi_d)^{\mathcal{I}(d<K)}\left\{\prod_{j=b}^{d}(1-p_j)(1-s_j)\right\} \\ &\times\left\{\psi_b(1-s_B)\xi_b(1-p_B)+\psi_b(1-s_B)(1-\xi_b)+1-\psi_b\right\}^{\mathcal{I}(d=K)}. \end{aligned} \tag{A.3}$$

Similarly, for U birds, the probability of CRR history h with at least one non-zero entry is:

$$
\begin{aligned}
L_h^U = \Bigg[& \sum_{b=1}^{f_h} \sum_{d=l_h}^{K} \beta_{b-1} \left\{ \prod_{j=b}^{d-1} \phi_j \right\} (1-\phi_d)^{\mathcal{I}(d<K)} \\
& \times \left\{ \left(\prod_{j=b}^{f_h^c-1} (1-p_j) \right) p_{f_h^c} \left(\prod_{j=f_h^c+1}^{d} (p_j')^{\mathcal{I}(x_{hj}^U=1 \text{ or } 3)} (1-p_j')^{\mathcal{I}(x_{hj}^U=0 \text{ or } 2)} \right) \right\} \\
& \times \left\{ \left(\prod_{j=b}^{f_h^c-1} (1-s_j) \right) \left(\prod_{j=f_h^c}^{d} (1-s_j) \right) + \left(1 - \prod_{j=b}^{f_h^c-1} (1-s_j) \right) \left(\prod_{j=f_h^c}^{d} (1-s_j') \right) \right\}^{\mathcal{I}(f_h^r=\text{NA})} \\
& \times \left\{ \left(\prod_{j=b}^{f_h^c-1} (1-s_j) \right) \left(\prod_{j=f_h^c}^{f_h^r-1} (1-s_j) \right) s_{f_h^r} \left(\prod_{j=f_h^r+1}^{d} (s_j')^{\mathcal{I}(x_{hj}^U=2 \text{ or } 3)} (1-s_j')^{\mathcal{I}(x_{hj}^U=0 \text{ or } 1)} \right) \right. \\
& \left. + \left(1 - \prod_{j=b}^{f_h^c-1} (1-s_j) \right) \left(\prod_{j=f_h^c}^{d} (s_j')^{\mathcal{I}(x_{hj}^U=2 \text{ or } 3)} (1-s_j')^{\mathcal{I}(x_{hj}^U=0 \text{ or } 1)} \right) \right\}^{\mathcal{I}(f_h^r\neq\text{NA})} \\
& \times \left\{ \psi_b(1-s_B)\xi_b(1-p_B) + \psi_b(1-s_B)(1-\xi_b) + 1 - \psi_b \right\}^{\mathcal{I}(d=K)} \Bigg]^{\mathcal{I}(l_h\leq K)} \\
\Bigg[& \sum_{b=1}^{\min(f_h,K)} \beta_{b-1} \left\{ \prod_{j=b}^{K-1} \phi_j \right\} \left\{ \prod_{j=b}^{K} (1-p_j) \right\}^{\mathcal{I}(f_h^c=\text{NA})} \\
& \times \left\{ \left(\prod_{j=b}^{f_h^c-1} (1-p_j) \right) p_{f_h^c} \left(\prod_{j=f_h^c+1}^{K} (p_j')^{\mathcal{I}(x_{hj}^U=1 \text{ or } 3)} (1-p_j')^{\mathcal{I}(x_{hj}^U=0 \text{ or } 2)} \right) \right\}^{\mathcal{I}(f_h^c\neq\text{NA})} \\
& \times \left\{ \left(\prod_{j=b}^{f_h^c-1} (1-s_j) \right) \left(\prod_{j=f_h^c}^{K} (1-s_j) \right) + \left(1 - \prod_{j=b}^{f_h^c-1} (1-s_j) \right) \left(\prod_{j=f_h^c}^{K} (1-s_j') \right) \right\}^{\mathcal{I}(f_h^r=\text{NA})} \\
& \times \left\{ \left(\prod_{j=b}^{f_h^c-1} (1-s_j) \right) \left(\prod_{j=f_h^c}^{f_h^r-1} (1-s_j) \right) s_{f_h^r} \left(\prod_{j=f_h^r+1}^{K} (s_j')^{\mathcal{I}(x_{hj}^U=2 \text{ or } 3)} (1-s_j')^{\mathcal{I}(x_{hj}^U=0 \text{ or } 1)} \right) \right. \\
& \left. + \left(1 - \prod_{j=b}^{f_h^c-1} (1-s_j) \right) \left(\prod_{j=f_h^c}^{K} (s_j')^{\mathcal{I}(x_{hj}^U=2 \text{ or } 3)} (1-s_j')^{\mathcal{I}(x_{hj}^U=0 \text{ or } 1)} \right) \right\}^{\mathcal{I}(f_h^r\neq\text{NA})} \\
& \times \psi_b \left\{ s_B^{\mathcal{I}(x_{h(K+1)}^U=1)} (1-s_B)^{\mathcal{I}(x_{h(K+1)}^U=0)} \right\}^{\mathcal{I}(f_h^c\neq NA)} \\
& \times (\xi_b p_B)^{\mathcal{I}(x_{h(K+2)}^U=1)} \left\{ \xi_b(1-p_B) \right\}^{\mathcal{I}(x_{h(K+2)}^U=2)} (1-\xi_b)^{\mathcal{I}(x_{h(K+2)}^U=0)} \\
& \times \eta_b^{\mathcal{I}(x_{h(K+3)}^U=1)} (1-\eta_b)^{\mathcal{I}(x_{h(K+3)}^U=0)} \Bigg]^{\mathcal{I}(l_h>K)} .
\end{aligned}
\tag{A.4}
$$

Details:

U individuals that have $l_h \leq K$ must have been caught during the NBS so they cannot have f_h^c =NA. Aside from that, the first two rows of the expression for L_h^U are the same as those of L_h^M.

- $$\left\{\left(\prod_{j=b}^{f_h^c-1}(1-s_j)\right)\left(\prod_{j=f_h^c}^{d}(1-s_j)\right)+\left(1-\prod_{j=b}^{f_h^c-1}(1-s_j)\right)\left(\prod_{j=f_h^c}^{d}(1-s_j')\right)\right\}^{\mathcal{I}(f_h^r=\text{NA})}:$$
  models the resight history of individuals that avoided resighting throughout their duration of stay in the NBS and have f_h^r=NA. These individuals can only be resighted after their first capture, but the probability of their resight history after f_h^c depends on whether they have used the feeders during the study and before f_h^c, which is not known. The probability that an individual has not used the feeders between their arrival time and time of first capture is $\left(\prod_{j=b}^{f_h^c-1}(1-s_j)\right)$ while the probability that they have used them at least once is $1-\left(\prod_{j=b}^{f_h^c-1}(1-s_j)\right)$. Conditional on not having recently used the feeders, the probability of avoiding resighting from f_h^c until d is $\left(\prod_{j=f_h^c}^{d}(1-s_j)\right)$ while conditional on having used the feeders at least once the corresponding probability is $\left(\prod_{j=f_h^c}^{d}(1-s_j')\right)$ so the unconditional probability of the resight history of these individuals is calculated using the law of total probability, as shown above.

- the resight histories for individuals with $f_h^r \neq$NA are again calculated using the law of total probability and the breeding history of these individuals is modelled in the same way as that for M birds with $l_h \leq K$.

- for individuals with $l_h > K$ d is known to be equal to K and they can be detected using the scanner only if they were caught at least once during the NBS and have

$f_h^c \neq$NA

The probability of the CRR history with all entries equal to 0 for U birds is:

$$L_0^U = \sum_{b=1}^{K} \sum_{d=b}^{K} \beta_{b-1} \left\{ \prod_{j=b}^{d-1} \phi_j \right\} (1-\phi_d)^{\mathcal{I}(d<K)} \left\{ \prod_{j=b}^{d} (1-p_j) \right\} \\ \times \left\{ \psi_b(1-\xi_b) + \psi_b \xi_b (1-p_B) + 1 - \psi_b \right\}^{\mathcal{I}(d=K)}. \quad \text{(A.5)}$$