Appendix A: Derivation of equations 4 and 7.

Deriving equations 4 and 7 requires some knowledge of ordinary differential equations, which many ecologists may not be familiar with. Therefore, we include a brief description here.

Equation 4. We begin with:

$$\frac{dp}{dt} = bp + c \tag{A.1}$$

because *a* in equation (1) is set to zero. One way of finding the function, *p*, in terms of *t*, that satisfies the above equation is to first define a new function *q* as,

$$q = p + c/b \tag{A.2}$$

Note that derivatives of *p* and *q* with respect to time are identical because neither *c* nor *b* varies with time. As a consequence, we can rewrite equation A.1 as,

$$\frac{dq}{dt} = bq \tag{A.3}$$

It can then be shown that the following equation for *q* satisfies A.3.

$$q(t) = ke^{bt} \tag{A.4}$$

where *k* is a constant. Reinserting this in equation A.2 and solving for *p*, we arrive at:

$$p(t) = ke^{bt} - c/b \tag{A.5}$$

Substituting $b = -(\gamma + \varepsilon)$, and $c = \gamma$ we get.

$$p(t) = ke^{-(\gamma+\varepsilon)t} + \frac{\gamma}{\gamma+\varepsilon} \tag{A.6}$$

Note that (A.6) differs only from equation 4 in the manuscript in that we have applied the initial condition that $p(0) = 0$ and solved for $k = -\frac{\gamma}{\gamma+\varepsilon}$.

Modeling declines. Equation A.6 could also be used to describe the dynamics of decline. For example, if the underlying colonization and extinction rates change such that the initial occupancy, p_i, is greater than the equilibrium values implied by the new colonization and extinction rates, then the dynamics of the decline can be modelled simply by substituting $k = p_i - \frac{\gamma}{\gamma+\varepsilon}$ into equation A.6. Extending this to look at different scenarios for the transient dynamics of *A* and *R* during a decline in two environments is straightforward.

Equation 7. We begin with the equation,

$$\frac{dp}{dt} = ap^2 + bp + c \tag{A.7}$$

with a<0, b>0 and c>0. Equation A.7 can be written in terms of its roots, $\lambda_{\prime}$ and $\lambda_{\prime\prime}$ as:

$$\frac{dp}{dt} = a(p - \lambda_{\prime})(p - \lambda_{\prime\prime}) \tag{A.8}$$

which can be split into the following partial fractions:

$$\frac{1}{dp/dt} = a^{-1}\left(\frac{A}{p-\lambda_{\prime}} + \frac{B}{p-\lambda_{\prime\prime}}\right) \quad \text{(A.8)}$$

where $A = 1/(\lambda_{\prime} - \lambda_{\prime\prime}) = -B$. We can then replace *A* with –*B* and rearrange A.8 to get:

$$\frac{a}{A}dt = \frac{dp}{p-\lambda_{\prime}} - \frac{dp}{p-\lambda_{\prime\prime}} \tag{A.9}$$

and integrate both sides to get,

$$\frac{a}{A}t + k = \ln|p - \lambda_{\prime}| - ln|p - \lambda_{\prime\prime}| = ln\left|\frac{p-\lambda_{\prime}}{p-\lambda_{\prime\prime}}\right| \tag{A.10}$$

Since c>0, this implies that p is slightly positive in the neighborhood of zero, therefore we know that in this neighborhood $\lambda_{\prime} < p < \lambda_{\prime\prime}$ and thus we can rewrite A.10 as,

$$\frac{a}{A}t + k = ln\left(\frac{p-\lambda_{\prime}}{\lambda_{\prime\prime}-p}\right) \tag{A.11}$$

which is equivalent to

$$Ke^{\frac{a}{A}t} = \frac{p-\lambda_{\prime}}{\lambda_{\prime\prime}-p} \tag{A.12}$$

where $K = e^k$. The initial condition $p(0) = 0$, imples that $K = \frac{-\lambda_{\prime}}{\lambda_{\prime\prime}}$. Inserting this value of *K* into A.12, as well as replacing *A* with $1/(\lambda_{\prime} - \lambda_{\prime\prime})$ leads to equation 7 in the main text.