APPENDIX C

MODEL EQUATIONS WITH CAUSAL DIAGRAM

Appendix C documents the model equations and associated parameter values used in our analyses, in the order of the modeling steps outlined in the paper. See Fig. C1 for an expanded diagram of the model, including parameters, symbols and equations from the following text.

Modeling steps

Estimating the number of gut piles eaten per eagle – The expected number of gut piles ingested per eagle ($\bar{C}$) increases to the maximum number of gut piles ingested (C^*) with availability of gut piles per Golden Eagle. Thus, across locations, expected scavenging increases with increasing number of gut piles per 100 km^2 (G), but decreases with increasing number of golden eagles per 100 km^2 ($GOEA$):

$$\bar{C} = \frac{\left(G/GOEA\right)^{\Psi}}{\left(G/GOEA\right)^{\Psi} + k_c^{\Psi}} C^* \qquad \text{(C.1)}$$

where k_c is the half-saturation constant, representing the available density of gut piles per eagle that would yield half the expected number of gut piles ingested per eagle in a 100 km^2 area. The exponent, ψ, is a shape parameter that describes how steep the threshold is between relatively low and high scavenging rates; $\psi > 1$ provides the Type III functional response shape.

We relied on expert judgment to describe the probability gut piles will be scavenged as a function of *G* and *GOEA*, from which we derive the expected number of gut piles ingested per eagle in any scenario of eagle and gut pile density. Given the considerable uncertainty about scavenging rates expressed among our experts, we define a range from low ($\bar{C}_{\min}$) to high number ($\bar{C}_{\max}$) of gut piles scavenged per eagle at any gut pile density and sampled from a uniform distribution between these bounds for each simulation so $\bar{C} \sim U(\bar{C}_{\min}, \bar{C}_{\max})$. For a given availability of gut piles per eagle, we determine $\bar{C}_{\min}$ by setting C^* equal to 1, k equal to 5 and ψ equal to 3 and for $\bar{C}_{\max}$ we set C^* equal to 5, k equal to 10 and ψ equal to 1.5. This range of gut piles scavenged per eagle aligns broadly with our expert's descriptions of the foraging relationships (see Appendix B, Fig. B8).

While the Type III functional response provides an expected (long-run population average) number of gut piles ingested per eagle, the number eaten by individual eagles will vary (we assume for modeling that gut piles are not divided between different eagles). We use a Poisson distribution to determine the discrete probability that an eagle ingests 0, 1 and up to the maximum possible number of gut piles an individual eagle could ingest in a month (C_{max}) given the expected average number ingested ($\bar{C}$). Because we assume there is a maximum possible number of gut piles ingested in a month by an eagle, we use the following application of the Poisson probability mass function to describe the probability that C gut piles are ingested per eagle as a function of the expected number eaten on average.

$$\hat{p}(C \mid \bar{C}) = \frac{\bar{C}^{C}}{C!} e^{-\bar{C}}, \qquad \text{if } \bar{C} < C_{\max}/2 \qquad \text{(C.2)}$$

$$\hat{p}(C\,|\,\overline{C}) = \frac{\left(C_{\max} - \overline{C}\right)^{C_{\max} - C}}{\left(C_{\max} - C\right)!} e^{-\left(C_{\max} - \overline{C}\right)}, \qquad \text{if } \overline{C} \geq C_{\max}/2$$

Based on expert opinion, we use five gut piles in a month as the value for C_{max}. Because we truncate the distribution, we must also normalize the probabilities so that they sum to one, thus yielding the probability of eating C gut piles, given an average of $\overline{C}$ gut piles eaten per eagle in the modeled area, or $p(C\,|\,\overline{C})$.

Estimating blood lead concentration per gut pile – The Cauchy distribution is a two-parameter distribution (mode, E_{mode}, and shape parameter, γ) that provides a probability for each of the blood lead concentration levels used in our expert elicitation. Specifically, the probability distribution of lead levels that describes the probability of an eagle having a peak concentration of E $\mu g/dL$ of lead in their blood per scavenge is:

$$\hat{p}(E) = \frac{\gamma}{\pi\left(E - E_{\text{mode}}\right)^2 + \gamma^2}. \qquad \text{(C.3)}$$

Like the probability of scavenging gut piles, we truncate the potential maximum increase in blood lead concentration per gut pile ingested so we must normalize the probability distribution. Thus the final probability distribution is:

$$p(E) = \frac{\hat{p}(E)}{\sum_{E=0}^{E_{\max}} \hat{p}(E)}. \qquad \text{(C.3a)}$$

Our experts expressed substantial uncertainty about the most likely increase in blood lead per scavenge of a lead-containing gut pile, E_{mode}, and we represent this uncertainty using a uniform distribution such that the mode concentration is: $E_{mode} \sim U(25,75)$. We set the maximum increase in blood lead concentration per scavenge, E_{max}, at 1000 $\mu g/dL$ and set the shape parameter, γ, to 25, to align with our experts' descriptions of blood lead concentration response to gut pile scavenging (see Appendix B, Fig. B9).

To account for the proportion of gut piles that contain no lead bullet fragments, we modify Eq. C.3a such that E is reduced by ϕ:

$$p(E) = (1-\phi)\frac{\hat{p}(E \mid E>0)}{\sum_{E=0}^{E_{\max}} \hat{p}(E)}, \quad \text{for } E > 0 \qquad \text{(C.3b)}$$

$$p(E=0) = (1-\phi)\frac{\hat{p}(E=0)}{\sum_{E=0}^{E_{\max}} \hat{p}(E)} + \phi, \text{ for } E = 0$$

We account for uncertainty in the frequency of gut piles containing no lead, when the animal was shot with lead ammunition, by drawing a uniform random number such that $\phi \sim U(10,50)$.

Estimating days between multiple gut piles scavenged – When eagles eat more than one carcass in a month we determine the maximum potential lag between meals:

$$L_{\max} = \left(\frac{30}{C^{*}-1}\right), \qquad \text{(C.4)}$$

where C^* is the maximum number of scavenging events per eagle for this location (determined for the iteration, see Eq. C.1). To model uncertainty in the average or expected lag time (in all simulations with $C^* \geq 2$), we draw a uniform integer between the minimum and maximum lag times (L_{min} and L_{max}) such that the average time between feedings, L, is $L \sim U(L_{min}, L_{max})$, where $L_{min} = 3$ days.

Estimating maximum blood lead by quantity of gut piles scavenged – Maximum blood lead concentration is a function of the average number of days between scavenge events (L) and a blood lead decay rate (D). The daily decay rate (D) in blood lead concentration is:

$$D = \exp\left(\frac{\ln(0.5)}{T_{half}}\right), \qquad \text{(C.5)}$$

where T_{half} is the average time in days it takes for the lead concentration in the blood to be reduced by half. To incorporate some uncertainty in this average, we draw a uniform random number such that $T_{half} = (10,20)$, indicating 10 to 20 days for the blood lead to be reduced by half.

Given the population average L and the estimate for D, we can calculate the average maximum concentration of lead in the blood during the month. The maximum amount of lead in the blood given the number of gut piles ingested (C) and the amount of lead exposure from each gut pile consumed (E) is:

$$Pb_{CE} = p(E)\sum_{i=1}^{C^*}\left(D^{L}\right)^{c^*-1}. \qquad \text{(C.6)}$$

Pb_{CE} is a probability distribution of maximum blood lead concentration for each potential quantity of gut piles consumed per eagle, from 1 to the maximum, C^*.

Estimating mortality by maximum blood lead – We use a saturating curve, much like a Michelis-Menton curve, such that mortality given the maximum concentration of lead in the blood during the month (Pb_{CE}) is:

$$M_{CE} = \frac{(Pb_{CE})^{\psi}}{k_m^{\psi} + (Pb_{CE})^{\psi}}, \quad \text{(C.7)}$$

where k_m is a half-saturation constant representing the concentration of lead in the blood that leads to a mortality rate of 50%. The exponent, ψ, is a shape parameter that describes how steep the threshold is between relatively low and high probability of mortality; the higher the ψ value, the steeper the threshold. The saturating curve is appropriate here because mortality cannot be larger than one and we expect that as the lead concentration increases, mortality will approach 1.

Our experts differed in their descriptions of the probabilistic relationship between maximum blood lead and mortality, M_{CE}, and we represent this uncertainty by using a uniform distribution such that the half-saturation is: $k_m \sim U(150,700)$. We set the maximum mortality rate at 1.0 and the shape parameter, ψ, to 2.5 for all runs, to align with our experts' descriptions of mortality rate per maximum blood lead concentration (see Appendix B, Fig. B10).

Integrating blood lead concentration and mortality: expected maximum blood lead and mortality for a site – Given the probability distribution for the number of gut piles scavenged ($p(C \mid \bar{C})$) and the probability distribution of blood lead concentration per gut pile consumed ($p(E)$), we can project the joint probability distribution describing the expected number of gut piles eaten and the blood lead concentration due to those gut piles, $p_{CE} \mid \bar{C}$. This simply the product of the two distributions, the expected maximum blood lead:

$$p_{CE} \mid \overline{C} = p(C \mid \overline{C}) p(E) \qquad \text{(C.8)}$$

To determine the expected mortality rate that accounts for multiple scavenge events, we simply need to multiply this joint probability distribution, $p_{CE} \mid \overline{C}$, by the mortality consequence of that combination, M_{CE}. The total expected mortality is thus influenced by the availability of carcasses per eagle and the amount of lead concentration increase per gut pile such that total expected mortality rate in the area (hunting unit) ($M \mid \overline{C}$) is:

$$M \mid \overline{C} = \sum_{C=0}^{C_{\max}} \sum_{E=0}^{E_{\max}} (p_{CE} \mid \overline{C})(M_{CE}) \qquad \text{(C.9)}$$

It follows that the number of golden eagles dying per area (hunting unit) is simply:

$$Deaths = GOEA * (M \mid \overline{C}) \qquad \text{(C.10)}$$

To estimate mortality rates at larger geographical scales, we sum the total deaths and divide by the total eagle abundance across all the units encompassed by the larger area.

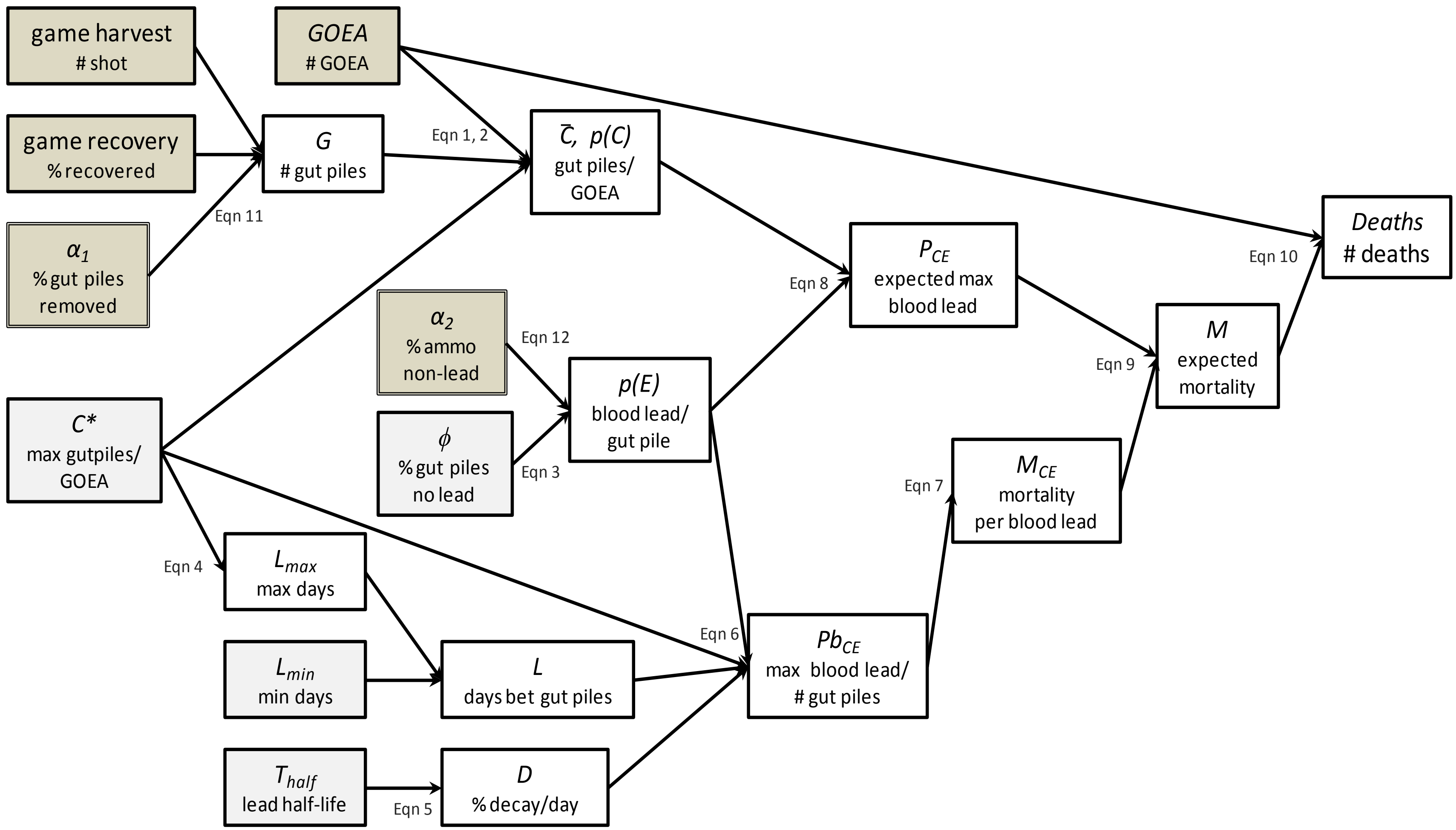
Incorporating mitigation – We represent the proportion of gut piles removed from the landscape as α_1 and the proportion of bullets that are non-lead as α_2. To incorporate gut pile removal, we modify Eq. C.1 such that G is reduced by α_1:

$$\overline{C} = \frac{\left(G \times \left(1-\alpha_1\right) / GOEA\right)^{\Psi}}{\left(G \times \left(1-\alpha_1\right) / GOEA\right)^{\Psi} + k^{\Psi}} C^* \qquad \text{(C.11)}$$

To incorporate replacement of lead with non-lead bullets, we reduce $p(E)$ (as calculated in Eq. C.3b) by α_2, the proportion of harvested big game animals shot with non-lead ammunition:

$$p(E) = \left(1-\alpha_2\right) \frac{\hat{p}(E \mid E>0)}{\sum_{E=0}^{E_{\max}} \hat{p}(E)}, \text{ for E} > 0 \qquad \text{(C.12)}$$

$$p(E=0) = \left(1-\alpha_2\right) \frac{\hat{p}(E=0)}{\sum_{E=0}^{E_{\max}} \hat{p}(E)} + \alpha_2, \text{ for E} = 0$$


game harvest
shot
GOEA
GOEA
game recovery
% recovered
G
gut piles
Eqn 1, 2
C̄, p(C)
gut piles/
GOEA
Eqn 11
α1
% gut piles
removed
Deaths
deaths
Eqn 10
P_CE
expected max
blood lead
Eqn 8
α2
% ammo
non-lead
Eqn 12
p(E)
blood lead/
gut pile
M
expected
mortality
Eqn 9
C*
max gutpiles/
GOEA
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no lead
Eqn 3
M_CE
mortality
per blood lead
Eqn 7
Eqn 4
L_max
max days
Eqn 6
Pb_CE
max blood lead/
gut piles
L_min
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days bet gut piles
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lead half-life
Eqn 5
D
% decay/day

FIG. C1. Causal diagram illustrating the parameters (boxes) and their relationships (arrows) in the golden eagle population model projecting number of eagles dying due to exposure to lead ammunition consumed from big game gut piles during one month of hunting season in a defined geographical area (hunting unit in our examples). Equation numbers are shown to the left of the calculated parameters. The predictive variables (darker shaded boxes) are set for each scenario and geographical unit modeled. These 'inputs' include (1) the total number of golden eagles in the specified region (*GOEA*), (2) two game harvest parameters for each harvest unit in that region - the number of big game animals shot during the fall hunting season and the proportion of game carcasses hunters retrieved and field-gutted, and (3) two mitigation scenario parameters: the proportion of hunters using non-lead ammunition (α_1), and the proportion of gut piles removed from the field by hunters (α_2). The last two parameters may be set to zero in status quo simulations without lead abatement mitigation. Additional input parameters essential to calculating lead ingestion and mortality (lighter shaded boxes) include the maximum number of gut piles scavenged per eagle per month (C^*), the minimum lag in days between multiple scavenging events (L_{min}), the half-life of lead in blood (T_{half}), and the proportion of gut piles without lead fragments (ϕ). The model uses these parameters to calculate all the variables in un-shaded boxes: (1) the number of gut piles per hunting unit (*G*) and the number of gut piles scavenged per eagle (*C*; Eqs. C.1-C.2), (2) the blood level increase per gut pile ingested (*E*; Eq. C.3), (3) the number of days between gut piles scavenged (*L*; Eq. C.4), (4) the daily decay rate of blood lead (*D*; Eq. C.5), (5) the maximum blood lead by total gut piles scavenged (Pb_{CE}; Eq. C.6), (6) the mortality rate per blood lead level (M_{CE}; Eq. C.7), (7) the expected maximum blood lead during the month (P_{CE}; Eq. C.8), and (7) the expected mortality rate (*M*; Eq. C.9) and the number of Golden Eagles deaths due to acute lead poisoning in a month (Eq. C.10). Mitigation affects the calculation of either gut piles per hunting unit (*G*; Eq. C.11) or blood level increase per gut pile ingested (*E*; Eq. C.12). Not depicted in the diagram are additional input parameters that determine the shape of the response curves for scavenging rate (gut piles/eagle), blood lead increase per gut pile, and mortality per blood lead (see Eqs. C.1-C.2, C.3, and C.7). The input parameters values have variance with specified distributions so the model output is a probability distribution of golden eagle deaths produced from repeated stochastic simulations. To estimate total deaths, calculations are completed for each geographical area (e.g., hunting unit) in an analysis area.