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APPENDIX A

Expectation and variance of overall probability of detection g across multiple independent areas or time periods.

Let *n* be the number of areas (e.g., sites, detection classes, turbine sets) or time periods (e.g., seasons, years) for which detection probabilities may differ and for which data (counts, detection probabilities, relative weights) are available. We estimate the total number of fatalities by calculating a pooled overall detection probability as the weighted average of detection probabilities over all sites and then applying Bayes' formula to the collection of sites considered as a single entity. The method for calculating a pooled g is described below.

Let G_k be a random variable for the conditional detection probability of a carcass that arrives at site *k* and let $\mathrm{E}G_k = g_k$ and $\mathrm{V}G_k = \sigma_k^2$.

Let $A_k = M_k/M$ be a random variable for the proportion of the total carcasses landing in site *i*, where $M = \sum_k M_k$ and M_k~Poisson(λ_k) are independent.

Then, let G be a random variable for the probability of detecting a carcass that arrives at one of the sites during the study period. We have: $G = \sum_k G_k A_k =$

$\sum_k P(\text{observe carcass}|\text{carcass arrives at site } k)P(\text{carcass arrives at site } k)$

Define $g = \mathrm{E}G = \mathrm{E}[\sum G_k A_k] = \sum \mathrm{E}[G_k A_k]$. But $\mathrm{E}[G_k A_k] = \mathrm{E}G_k \cdot \mathrm{E}A_k$ because G_k and A_{ik} are independent. Define $\mathrm{E}G_k = g_k$ and note that $\mathrm{E}A_k = \mathrm{E}[M_k/M] \cong \frac{\lambda_k}{\lambda}\left(1 + \frac{1}{\lambda}\right) - \mathrm{cov}(M_k, M)/\lambda^2$ (Mood et al. 1974). But $M = M_k + M'$, where $M' = \sum_{j \neq k} M_j$ and the M_j are independent. Thus,

$\mathrm{cov}(M_k, M) = \mathrm{cov}(M_k, M_k + M') = \mathrm{cov}(M_k, M_k) + \mathrm{cov}(M_k, M') = \mathrm{V}M_k + 0 = \lambda_k$, so $\mathrm{E}A_k \cong \frac{\lambda_k}{\lambda}$, which we'll define as a_k, or the expected proportion of carcasses landing at site *k*.

Therefore, we have:

$$\mathrm{E}G \cong \sum g_k \lambda_k / \lambda$$

The variance of G is given by

$\mathrm{V}G = \mathrm{V}[\sum G_k A_k] = \mathrm{V}[\sum G_k M_k / M] \cong \sum a_k^2 \sigma_k^2 + 1/\lambda \sum a_k (g_k^2 + \sigma_k^2) - g^2/\lambda$ (Mood et al. 1974), which can be shown in several steps. For $n = 2$, $\mathrm{V}[A_1 G_1 + A_2 G_2] = \mathrm{V}\left[\frac{M_1 G_1 + M_2 G_2}{M}\right]$ $\cong (\mu_X/\mu_M)^2 \left(\frac{\sigma_X^2}{\mu_X^2} + \frac{\sigma_M^2}{\mu_M^2} - \frac{2\,\mathrm{cov}(X,M)}{\mu_X \mu_M}\right)$, where $X = M_1 G_1 + M_2 G_2$ and the μ and σ^2 terms represent the means and variances of their subscripted variables. The components of the expression are calculated below.

1. $\mu_M = \sigma_M^2 = \lambda$

2. $\mu_X = \lambda_1 g_1 + \lambda_2 g_2$

3. $\sigma_X^2 = \mathrm{V}[M_1 G_1 + M_2 G_2] = \mathrm{V}[M_1 G_1] + \mathrm{V}[M_2 G_2]$, since carcasses both arrive independently and are observed independently at different sites, and $\mathrm{V}[M_k G_k] = \mathrm{E}^2 M_k \cdot \mathrm{V}g_k + \mathrm{E}^2 g_k \cdot \mathrm{V}M_k + \mathrm{V}g_k \cdot \mathrm{V}M_k = \lambda_k^2 \sigma_k^2 + g_k^2 \lambda_k + \sigma_k^2 \lambda_k$ for $k = 1, 2$, so $\mathrm{V}[X] = \sum \lambda_k^2 \sigma_k^2 + \sum \lambda_k (g_k^2 + \sigma_k^2)$

4. $\mathrm{cov}[X, M] = \mathrm{cov}[M_1 G_1 + M_2 G_2, M] = \mathrm{cov}[M_1 G_1, M_1 + M_2] + \mathrm{cov}[M_2 G_2, M_1 + M_2]$ $= \mathrm{cov}[M_1 G_1, M_1] + \mathrm{cov}[M_1 G_1, M_2] + \mathrm{cov}[M_2 G_2, M_1] + \mathrm{cov}[M_2 G_2, M_2]$. The middle two terms are zero because the sites are independent. The first and last terms are calculated as $\mathrm{cov}[M_k G_k, M_k] = \mathrm{E}[G_k M_k^2] - \mathrm{E}[M_k G_k]\mathrm{E}[M_k] = \mathrm{E}G_k \cdot \mathrm{E}M_k^2 - \mathrm{E}G_k \mathrm{E}^2 M_k$ $= \mathrm{E}G_k\left(\mathrm{E}M_k^2 - \mathrm{E}^2 M_k\right) = g_k \lambda_k$, so $\mathrm{cov}[X, M] = g_1 \lambda_1 + g_2 \lambda_2$.

Combining the terms into the original expression gives:

$$\mathrm{V}G \cong \left(\frac{\lambda_1 g_1+\lambda_2 g_2}{\lambda}\right)^2 \left(\frac{\lambda_1^2\sigma_1^2+\lambda_2^2\sigma_2^2+\lambda_1(g_1^2+\sigma_1^2)+\lambda_2(g_2^2+\sigma_2^2)}{(\lambda_1 g_1+\lambda_2 g_2)^2} + \frac{\lambda}{\lambda^2} - 2\frac{g_1\lambda_1+g_2\lambda_2}{(g_1\lambda_1+g_2\lambda_2)\lambda}\right) =$$

$\frac{\lambda_1^2\sigma_1^2+\lambda_2^2\sigma_2^2+\lambda_1(g_1^2+\sigma_1^2)+\lambda_2(g_2^2+\sigma_2^2)}{\lambda^2} - \frac{g^2}{\lambda}$, which can be rewritten as:

$$\mathrm{V}G \cong \sum a_k^2\sigma_k^2 + \frac{\sum a_k(g_k^2+\sigma_k^2) - g^2}{\lambda}$$

By mathematical induction, the sums can be taken over $k = 1, \ldots, n$.

The calculations are identical if we are considering n time periods instead of n areas.