Appendix B - Full results from all simulations

² Parameter Bias

The estimates of μ were consistently unbiased, but the precision of these estimates depended almost entirely on the length of the observation period and very little on the number of total observations (Figure B1).

In most simulated scenarios, estimates of process and measurement error variance were 6 also unbiased. However, when measurement error variance was relatively quite small 7 $(\sigma^2/\tau^2 = 5)$, and there were less than one observation per time step, we tended to 8 overestimate measurement error variance, probably because that parameter was difficult to 9 identify. This in turn led to an underestimate of the process error variance. This bias was 10 reduced if there were one or more observations per time step. Similarly, when the process 11 error variance was relatively quite small ($\sigma^2/\tau^2 = 0.05$), we tended to overestimate the 12 process error variance and underestimate the measurement error variance, particularly if 13 the total number of observations was low, regardless of the length of the time series 14 (Figures B2 and B3). 15

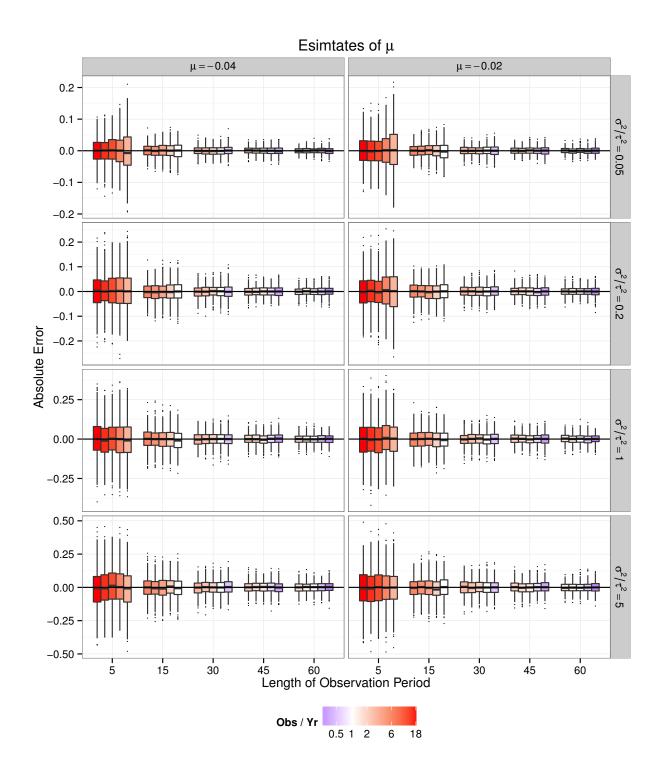


Figure B1: Absolute error (estimated value - true value) for estimates of the rate of decline grouped by length of observation period and observations per time step. Each column corresponds to different simulated values of μ , and each row corresponds to a different process to non-process error variance ratio. The dark line depicts the median, the boxes extend between the 25th and 75th quantiles, and the whiskers extend to a maximum of 1.5 times the inter-quartile difference.

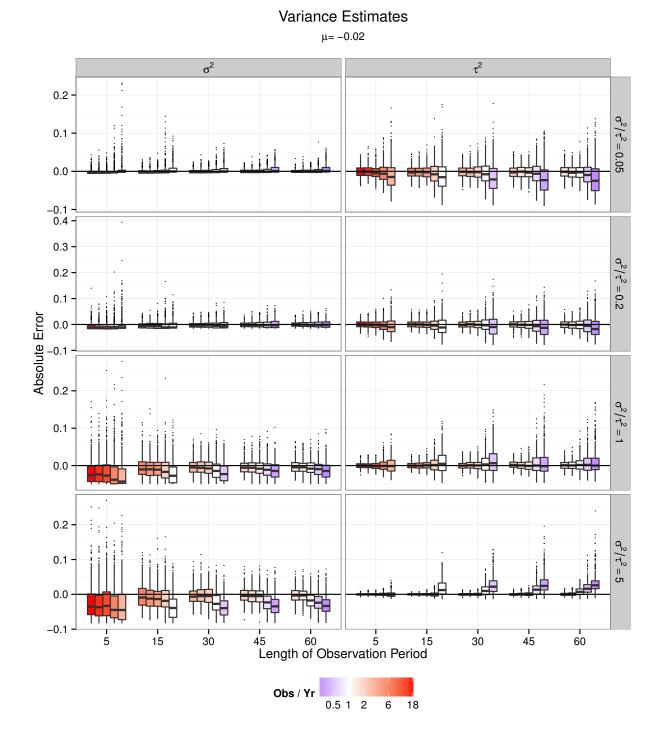


Figure B2: Absolute error (estimated value - true value) for estimates of the process (σ^2) and measurement (τ^2) error variances, grouped by length of observation period and observations per time step, when the simulated value of μ is -0.02. Each row corresponds to a different process to non-process error variance ratio. The dark line depicts the median, the boxes extend between the 25th and 75th quantiles, and the whiskers extend to a maximum of 1.5 times the inter-quartile difference.

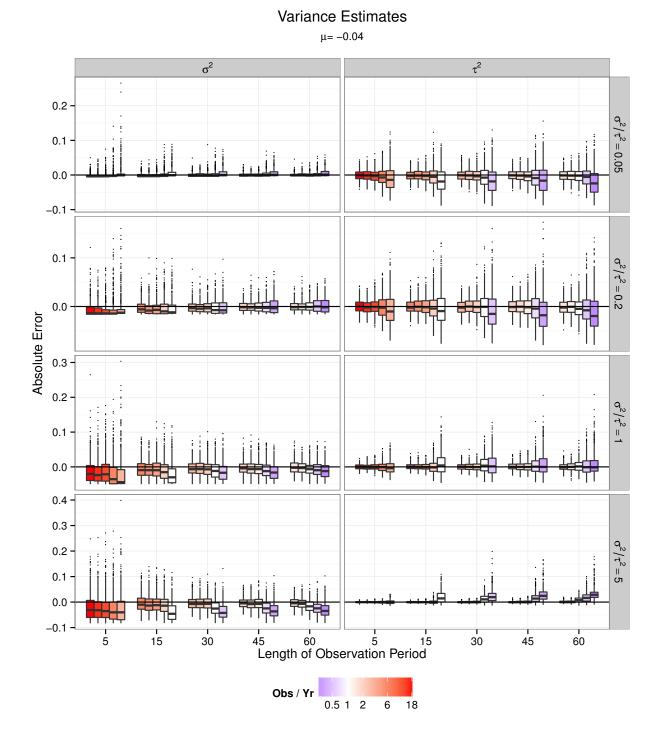


Figure B3: Absolute error (estimated value - true value) for estimates of the process (σ^2) and measurement (τ^2) error variances, grouped by length of observation period and observations per time step, when the simulated value of μ is -0.04. Each row corresponds to a different process to non-process error variance ratio. The dark line depicts the median, the boxes extend between the 25th and 75th quantiles, and the whiskers extend to a maximum of 1.5 times the inter-quartile difference.

¹⁶ Parameter Precision

¹⁷ The effects of observation period length and total observations on parameter precision were ¹⁸ very similar across both values of μ that were simulated (Figure B4, and see Figure 1 in ¹⁹ manuscript).

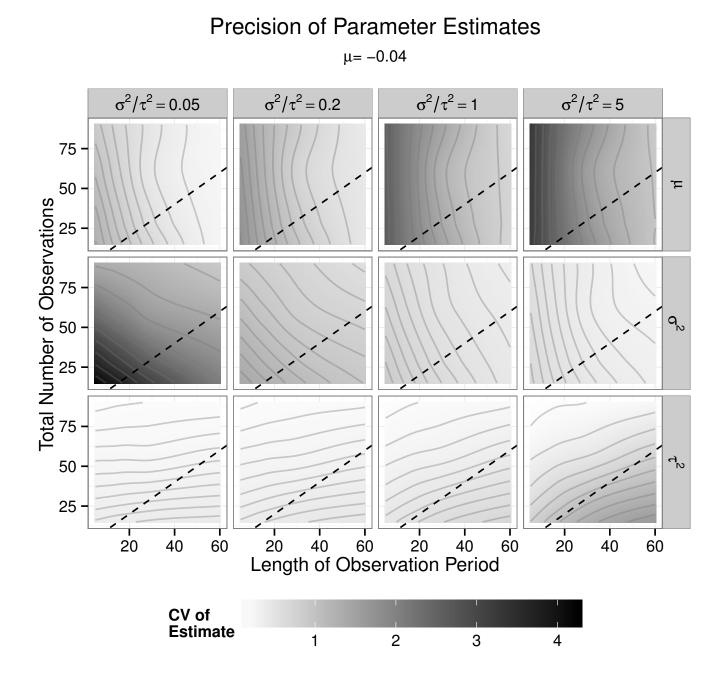


Figure B4: Contour plots of the coefficient of variation of the parameter estimates for rate of decline (top row, μ), process error variance (middle row, σ^2) and measurement error variance (bottom row, τ^2), when the true value of μ is -0.04. Each column corresponds to a different process to measurement error variance ratio. The dashed (- -) line corresponds to scenarios where one observation is taken every time step.

20 Probability of Quasi-Extinction

The probabilities of quasi-extinction across both time horizons (30 and 50 years) were estimated with little bias for all three levels of decline, all combinations of observation period length and number of total observations and both long-term rates of decline (Figures B5 and B6).

We examined the effect of the number of observations and the length of the 25 observation period on the precision of estimates for the risk of quasi-extinction by 26 comparing the precision to a reference case: a thirty year time series with one observation a 27 year. This was chosen because it depicts a realistic dataset with which a biologist of 28 manager would try to estimate the risk of quasi-extinction (Holmes et al. 2007). In general, 29 precision improves as the length of the observation period increases. The precision 30 decreases, sometimes rapidly, once the length of the observation period is smaller than 20 31 time steps, across all simulation scenarios (Figures B7 and B8). 32

μ=-0.02 $\sigma^2/\tau^2 = 1$ $\sigma^2/\tau^2 = 0.05$ $\sigma^2/\tau^2 = 0.2$ $\sigma^2/\tau^2 = 5$ Perc.Decl. = 30 0.00 -0.25 -0.50 **Obs** / **Yr** 18 -0.75 0.5 Absolute Error Perc.Decl. = 50 6 0.0 2 -0.5 1 1.0 0.5 Perc.Decl. 0.5 0.0 . = 80 -0.5 15 30 45 5 45 . 45 5 60 15 30 60 5 15 30 45 60 5 15 30 60 Length of Observation Period Estimating a Decline over 30 Years μ=-0.04 $\sigma^2/\tau^2 = 0.05$ $\sigma^2/\tau^2 = 0.2$ $\sigma^2/\tau^2 = 1$ $\sigma^2/\tau^2 = 5$ 0.00 Perc.Decl. = 30 -0.25 -0.50 -0.75 **Obs** / **Yr** 18 -1.00 Absolute Error Perc.Decl. = 50 0.00 6 -0.25 -0.50 2 -0.75 1 0.5 Perc.Decl 0.5 0.0 = 80 -0.5 -5 30 15 45 5 15 30 45 60 . 5 15 30 45 60 5 15 45 60 30 60 Length of Observation Period

Estimating a Decline over 30 Years

Figure B5: Absolute error of estimates of the probability of quasi-extinction within 30 years, grouped by the length of the observation period and observations per year. The rows correspond to different quasi-extinction thresholds (30, 50 and 80% declines) and the columns correspond to different process to measurement error variance ratios. The top and bottom set of box plots correspond to different simulated values of μ . The dark line depicts the median, the boxes extend between the 25th and 75th quantiles, and the whiskers extend to a maximum of 1.5 times the inter-quartile difference.

μ=-0.02 $\sigma^2/\tau^2 = 1$ $\sigma^2/\tau^2 = 0.05$ $\sigma^2/\tau^2 = 0.2$ $\sigma^2/\tau^2 = 5$ 0.00 Perc. Decl. = 30 -0.25 -0.50 **Obs** / **Yr** 18 -0.75 -1.00 Absolute Error 100 Perc.Decl. = 50 di l'O 0.0 6 -0.3 2 -0.6 1 0.5 Perc.Decl. = 80 0.5 0.0 -0.5 5 45 . 45 5 15 30 . 45 60 15 30 60 5 15 30 45 60 5 15 30 60 Length of Observation Period Estimating a Decline over 50 Years μ=-0.04 $\sigma^2/\tau^2 = 0.05$ $\sigma^2/\tau^2 = 0.2$ $\sigma^2/\tau^2 = 1$ $\sigma^2/\tau^2 = 5$ 0.00 Perc.Decl. = 30 -0.25 -0.50 . -0.75 **Obs** / **Yr** 18 -1.00 . i i i i 0.00 В 0.25 Holo-0.57 Volute Volute Volute Volute Perc.Decl. = 50 6 2 1 0.5 Perc.Decl. = 80 0.0 -0.3 -0.6 5 15 30 . 45 60 5 15 30 . 45 60 5 15 30 45 60 5 15 . 30 45 60 Length of Observation Period

Estimating a Decline over 50 Years

Figure B6: Absolute error of estimates of the probability of quasi-extinction within 50 years, grouped by the length of the observation period and observations per year. The rows correspond to different quasi-extinction thresholds (30, 50 and 80% declines) and the columns correspond to different process to measurement error variance ratios. The top and bottom set of box plots correspond to different simulated values of μ . The dark line depicts the median, the boxes extend between the 25th and 75th quantiles, and the whiskers extend to a maximum of 1.5 times the inter-quartile difference.

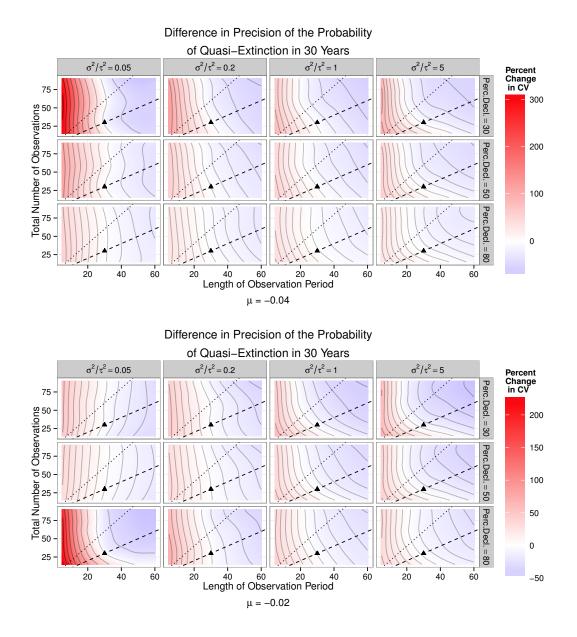


Figure B7: Contour plots of the percent improvement to the coefficient of variation (CV) of the estimates of the probability of a 50% decline in 30 years, under two simulated rates of decline (top and bottom sets of plots), compared to a reference case of a 30 year observation period with one observation each year (marked with a \blacktriangle). The white contour area corresponds to the same level of precision as the reference case. Areas in blue have a more precise estimate of P_e than the reference case, while areas in red have a less precise estimate. The dashed (- -) and dotted (...) straight lines correspond to scenarios where one and two observation(s) are taken every time step. Each column corresponds to a different process to measurement error variance ratio.

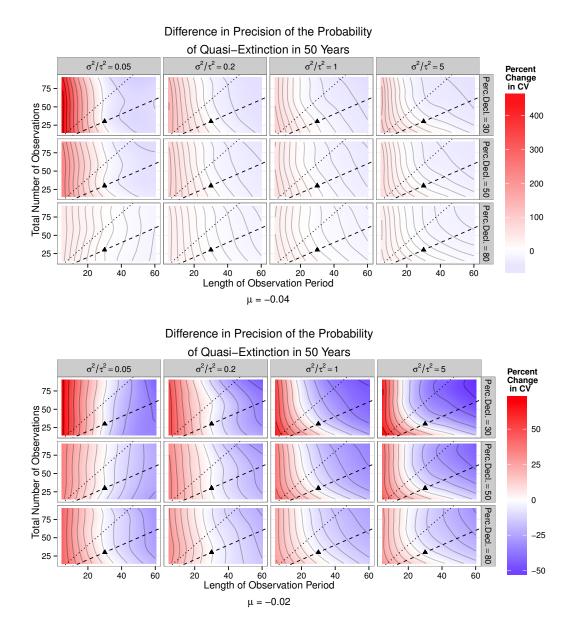


Figure B8: Contour plots of the percent improvement to the coefficient of variation (CV) of the estimates of the probability of a 50% decline in 50 years, under two simulated rates of decline (top and bottom sets of plots), compared to a reference case of a 30 year observation period with one observation each year (marked with a \blacktriangle). The white contour area corresponds to the same level of precision as the reference case. Areas in blue have a more precise estimate of P_e than the reference case, while areas in red have a less precise estimate. The dashed (- -) and dotted (...) straight lines corresponds to a different process to measurement error variance ratio.

33 References

- ³⁴ Holmes, E. E., Sabo, J. L., Viscido, S. V., and Fagan, W. F. (2007). A statistical approach
- to quasi-extinction forecasting. *Ecology Letters*, 10(12):1182–1198.