

# Appendix A - Comparison of Estimation Methods

We compared several parameter estimation procedures, including maximum likelihood estimates obtained from the Kalman filter and EM algorithm, using the MARSS package (Holmes and Ward 2010), first difference restricted maximum likelihood estimates (REML 1) (Dennis et al. 2010) and second difference restricted maximum likelihood estimates (REML 2) (Humbert et al. 2009). All estimates were obtained using R software (R Development Core Team 2009). The comparison of estimation methods was carried out on a subset of the simulations; only simulated time series of 30 time steps were used, with total observations equal to 30, 60 and 90, and only one mean rate of decline ( $\mu = -0.04$ ).

Based on time series of 30 time steps, the maximum likelihood estimates provided by the MARSS package were the least biased and most precise, compared to REML estimates based on first and second differences (Figure A1), particularly for the mean population trend,  $\mu$ . REML estimates based on the first differences of the observations were biased low for the mean trend and biased high for the process error variance, at least when the true process error variance was relatively small. This bias disappears when we consider the REML estimates based on the second differences of the observations, but they are still more variable than the MARSS estimates for the mean population trend, and often comparable in bias and variability for other parameters to the MARSS estimates. Therefore, we used the MARSS package to obtain estimates for all other results.

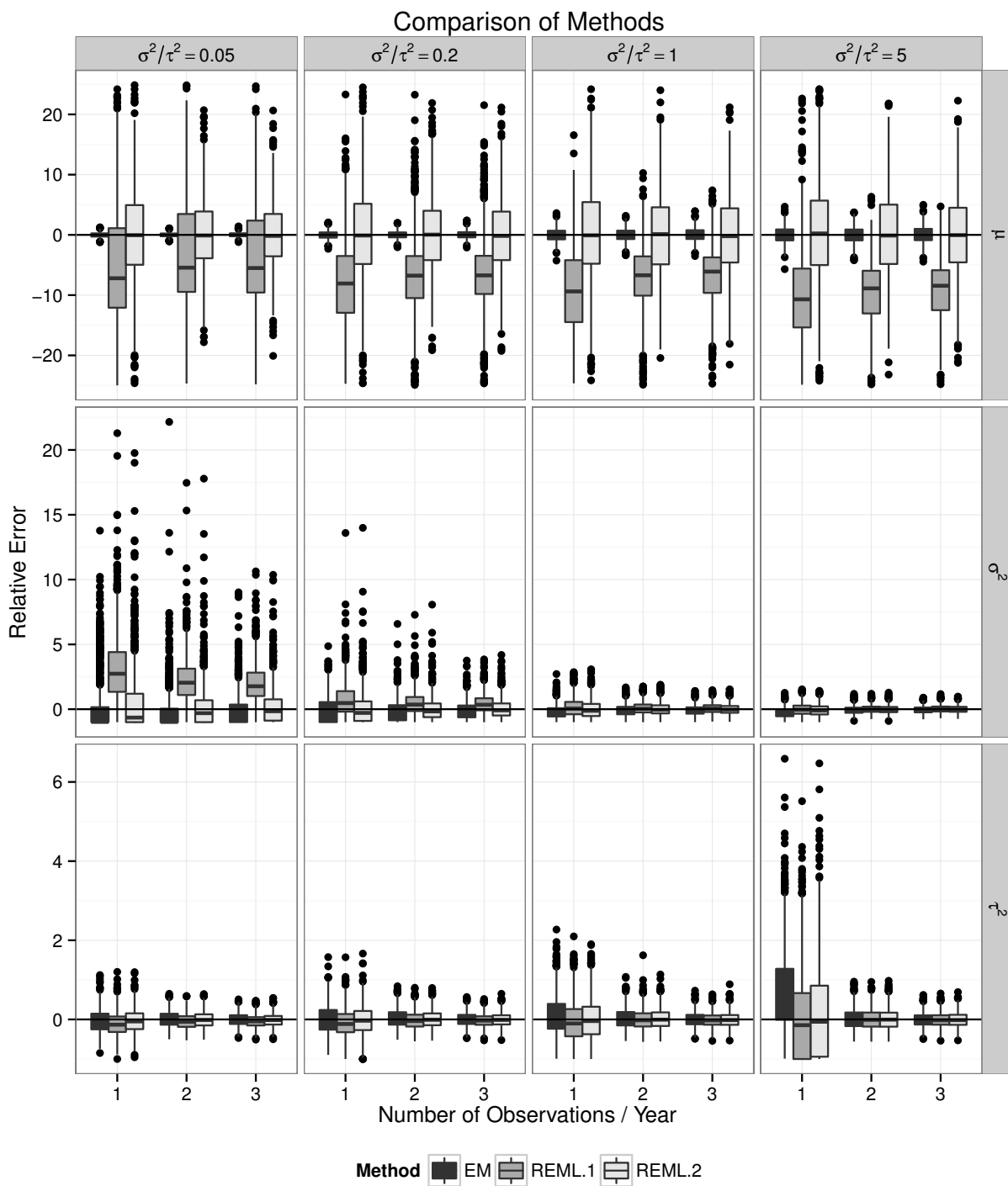


Figure A1: Boxplots of the relative error of the parameter estimates under three different estimation procedures: maximum likelihood using an EM algorithm (dark gray boxplots), restricted maximum likelihood based on first differences of the observations, REML 1, (light gray boxplots) and restricted maximum likelihood based on second differences of the observations, REML 2 (white boxplots). N refers to the number of observations simulated at each of 30 time steps.

## 20 References

- 21 Dennis, B., Ponciano, J. M., and Taper, M. L. (2010). Replicated sampling increases  
22 efficiency in monitoring biological populations. *Ecology*, 91(2):610–620.
- 23 Holmes, E. E. and Ward, E. J. (2010). *Analysis of multivariate time-series using the*  
24 *MARSS package*. NOAA Fisheries, Northwest Fisheries Science Center, 2725 Montlake  
25 Blvd E., Seattle, WA 98112, version 2.1 edition.
- 26 Humbert, J. Y., Mills, L. S., Horne, J. S., and Dennis, B. (2009). A better way to estimate  
27 population trends. *Oikos*, 118(12):1940–1946.
- 28 R Development Core Team (2009). *R: A Language and Environment for Statistical*  
29 *Computing*. R Foundation for Statistical Computing, Vienna, Austria. ISBN  
30 3-900051-07-0.