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APPENDIX A. Theoretical Basis for Estimating Rare-Event Bycatch Using a Bayesian Approach.

For a single species (e.g., leatherbacks), we use a Poisson likelihood function to model the stochastic dependence of x_i, the number of observed takes in year i, on θ, the per-set take rate parameter, and n_i, the number of observed sets in year i:

$$f(x_i \,|\, \theta, n_i) = e^{-\theta n_i} \frac{(\theta n_i)^{x_i}}{x_i!}, \tag{A.1}$$

where $\theta n_i = \lambda_i$ is the Poisson rate (mean) parameter. Previous studies have also used the Poisson distribution to model bycatch (NMFS 2004, Pradhan and Leung 2006, Gardner et al. 2008, Murray 2009, 2011) because it can characterize data in which each observation has a high probability for a zero count, a small probability for a count of one, and an infinitesimal probability for a count of two or more. The DGN fishery data have these Poisson characteristics plus one more: a mean per-set take rate (2.944×10^{-3}) roughly equal to the variance (2.936×10^{-3}). A strong positive correlation between the numbers of observed takes and sets per year (Pearson's r=0.672; p=0.001; Figures 1 and 2) supports including the number of sets in the model. We assume statistical independence of all sets and takes.

We use a conjugate gamma prior distribution for θ:

$$p(\theta) \propto e^{-\beta\theta}\theta^{\alpha-1}, \tag{A.2}$$

which constrains θ to be positive. After applying Bayes' rule, the posterior density,

$$p(\theta \,|\, n_i, x_i) \propto f(x_i \,|\, \theta, n_i)p(\theta) \tag{A.3}$$

$$\propto e^{-(\beta+n_i)\theta}\theta^{\alpha+x_i-1}, \tag{A.4}$$

is also a gamma distribution, $\Gamma\left(\alpha + x_i, \beta + n_i\right)$, with a form that suggests interpreting α and β as the prior numbers of observed takes and sets from previous years, respectively, before observing the current year's sample of x_i takes in n_i sets.

Following this interpretation, a noninformative prior could be specified by assigning $\alpha = 0$ and $\beta = 0$, yielding

$$p(\theta) = \theta^{-1},\ 0 < \theta < \infty, \tag{A.5}$$

which is diffuse and improper (does not integrate over the support). This prior reflects ignorance about θ before observing the data, and places the greatest weight on values near zero. The resulting posterior,

$$p(\theta \,|\, n_i, x_i) \propto e^{-\theta n_i}\theta^{x_i-1}, \tag{A.6}$$

bears formal similarity to the likelihood function, but now summarizes reasonable beliefs about θ in light of the current observation of x_i. The posterior mean, $\mu_\theta = \frac{x_i}{n_i}$, and variance, $\sigma_\theta^2 = \frac{x_i}{n_i^2}$, are formally identical to the maximum likelihood estimator and variance of the maximum likelihood estimator of θ in the classical Poisson model, but are subject to a different interpretation under the Bayesian paradigm.

To specify an informative prior, we could assign $\alpha = x_p$ and $\beta = n_p$, where x_p and n_p are the respective numbers of observed takes and sets in all previous years p:

$$p(\theta) \propto e^{-\theta n_p} \theta^{x_p - 1}. \tag{A.7}$$

The corresponding posterior,

$$p(\theta \mid n_i, x_i) \propto e^{-\theta(n_p + n_i)} \theta^{x_p + x_i - 1}, \tag{A.8}$$

has a mean $\mu_\theta = \frac{x_p + x_i}{n_p + n_i}$ and variance $\sigma_\theta^2 = \frac{x_p + x_i}{(n_p + n_i)^2}$.

The posterior predictive distribution (PPD) for the number of unobserved takes, $y_i - x_i$, is derived from the Poisson likelihood function and the posterior for θ:

$$p(y_i - x_i \mid N_i, n_i, x_i) = \int_\theta p(y_i - x_i \mid \theta, N_i - n_i) p(\theta \mid n_i, x_i) d\theta, \tag{A.9}$$

where y_i and N_i are the total (observed + unobserved) numbers of takes and sets in year i, respectively. This is a negative binomial distribution, $Negbin(\alpha + y_i - x_i, \frac{\beta + n_i}{N_i - n_i})$, where α and β are again the numbers of observed takes and sets from all previous years (Gelman et al. 2004). This PPD reflects posterior uncertainty in θ and in unobserved experience. Adding x_i produces a PPD for y_i. Whereas a posterior distribution supports inference about a parameter in the likelihood function (in our case, θ, a bycatch rate), a posterior *predictive* distribution supports predictive statements about the output of the likelihood function (in our case, $y_i - x_i$, the unobserved bycatch count). A PPD may be specified for any amount of fishing effort based on past numbers of observed sets and takes, regardless of whether the effort occurred in the past or has yet to occur in the future. This assumes that the same probability model holds under different years and conditions, which seems reasonable given that the distribution of these rare-event takes over 20 years appears to follow a Poisson distribution. The PPD can be used to produce range or point estimates of bycatch for the specified level of effort.

To model the number of observed deaths, w_i, we specify a binomial likelihood function which quantifies the stochastic dependence of w_i on x_i and a conditional mortality parameter, q (i.e., given a take of this species occurs, the probability that the animal dies):

$$f(w_i \mid x_i, q_s) = \frac{x_i!}{w_i!(x_i - w_i)!} q^{w_i} (1-q)^{x_i - w_i}. \tag{A.10}$$

A noninformative, conjugate prior of $Beta(1, 1)$ for q results in a posterior distribution of the form $Beta(1 + w_p, 1 + x_p - w_p)$ (Chapter 2 in Gelman et al. (2004)). Over 20 years, $w_p = 14$ deaths and $x_p = 24$ takes for leatherbacks, and $w_p = 1$ death and $x_p = 4$ takes for humpbacks (the fisher-reported mortality is conservatively treated as a take for purposes of estimating the conditional mortality rate for humpbacks). Throughout our analyses, we use the posteriors $Beta(15, 11)$ for leatherbacks and $Beta(2, 4)$ for humpbacks.

The PPD for the number of unobserved deaths, $z_i - w_i$, can be constructed using $Binomial$(PPD for $y_i - x_i, q$). Adding w_i to this distribution produces a PPD for z_i, the total number of deaths for that species in year i.

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