APPENDIX B: MODELING OF THE MULTIPLE DETECTION METHODS DESIGN.

Under a multiple detection methods sampling design, we define two detection methods: a primary method (M1), which allows for false positives and therefore generates ambiguous detections, and a secondary method (M2) that always provides unambiguous detections. Here, the I sites are visited T times using the detection method M1 and S times using the method M2. For the i^{th} site, the observation obtained with M1 on the t^{th} visit is denoted as y_{it} and the observation obtained with M2 on the s^{th} visit is denoted as w_{is}. Both y_{it} and w_{is} are binary variables taking value 1 when the species is detected and 0 otherwise. We define three detection parameters: (i) p_{11} is the probability of detecting the species with method M1 given that the site is occupied; (ii) p_{10} is the probability of erroneously detecting the species with method M1 given that the site is unoccupied; and (iii) r_{11} is the probability of detecting the species with method M2 given that the site is occupied. Full conditional observation probabilities are shown in Table B1.

The hierarchical construction of the likelihood for these data is:

$z_i \sim Bernoulli(\psi)$

$(y_{it}|z_i) \sim Bernoulli(z_i p_{11} + (1 - z_i)p_{10})$

$(w_{is}|z_i) \sim Bernoulli(z_i r_{11})$

The likelihood is also written as:

$$L(\psi, p_{11}, p_{10}, r_{11} \mid \boldsymbol{y}, \boldsymbol{w}) \propto \prod_{i=1}^{I}\Big(\Big[\psi \times \Big\{\prod_{t=1}^{T} p_{11}^{y_{it}}(1-p_{11})^{1-y_{it}}\Big\} \times \Big\{\prod_{s=1}^{S} r_{11}^{w_{is}}(1-r_{11})^{w_{is}}\Big\}\Big] + \Big[(1-\psi) \times \Big\{\prod_{t=1}^{T} p_{10}^{y_{it}}(1-p_{10})^{1-y_{it}}\Big\} \times I(w_{is}=0)\Big]\Big)$$

where $I(w_{is} = 0)$ is an indicator function that equals 1 if $w_{is} = 0$ is satisfied, 0 otherwise.

TABLE B1. Observation matrices of the multiple detection method model of the site confirmation approach. The variables and parameters are defined as follows: *z* is the true occupancy state of a site; *y* is the detection/non-detection data obtained from the ambiguous detection method; *w* is the detection/non-detection data obtained from the unambiguous detection method; $p_{10}$is the conditional probability of site-level false positive and p_{11} is the conditional probability of site-level detection, both for the ambiguous method; and r_{11} is the conditional probability of site-level detection for the unambiguous method.

True State	Ambiguous detection method		Unambiguous detection method	
	$y = 0$	$y = 1$	$w = 0$	$w = 1$
$z = 0$	$1 - p_{10}$	p_{10}	1	0
$z = 1$	$1 - p_{11}$	p_{11}	$1 - r_{11}$	r_{11}