Heidi Swanson, Martin Lysy, Michael Power, Ashley Stasko, Jim Johnson, and James Reist. 2014. A new probabilistic method for quantifying n-dimensional ecological niches and niche overlap. *Ecology*

APPENDICES

Appendix B. Calculation of niche overlap.

Appendix B. Calculation Niche Overlap

Here we detail the Monte Carlo algorithm for estimating the overlap metric $\mathrm{O}\binom{A}{B}$ for Normally distributed species $X_A \sim \mathcal{N}(\mu_A, \Sigma_A)$ and $X_B \sim \mathcal{N}(\mu_B, \Sigma_B)$.

1. Simulate M iid draws $X_A^{(1)}, \ldots, X_A^{(M)}$ from the distribution $\mathcal{N}(\mu_A, \Sigma_A)$ of species A.
2. For each draw, let $I_m = 1$ if $X_A^{(m)}$ is inside $\mathrm{N_R}(B)$ and $I_m = 0$ otherwise. This amounts to checking whether or not $X_A^{(m)}$ satisfies the quadratic inequality **??**.
3. The overlap metric is approximated by $\hat{\mathrm{O}}\binom{A}{B} = \frac{1}{M} \sum_{m=1}^{M} I_m$, an unbiased estimate of the true niche overlap with variance decreasing to 0 as $M \to \infty$.

The procedure is illustrated in Figure B1.

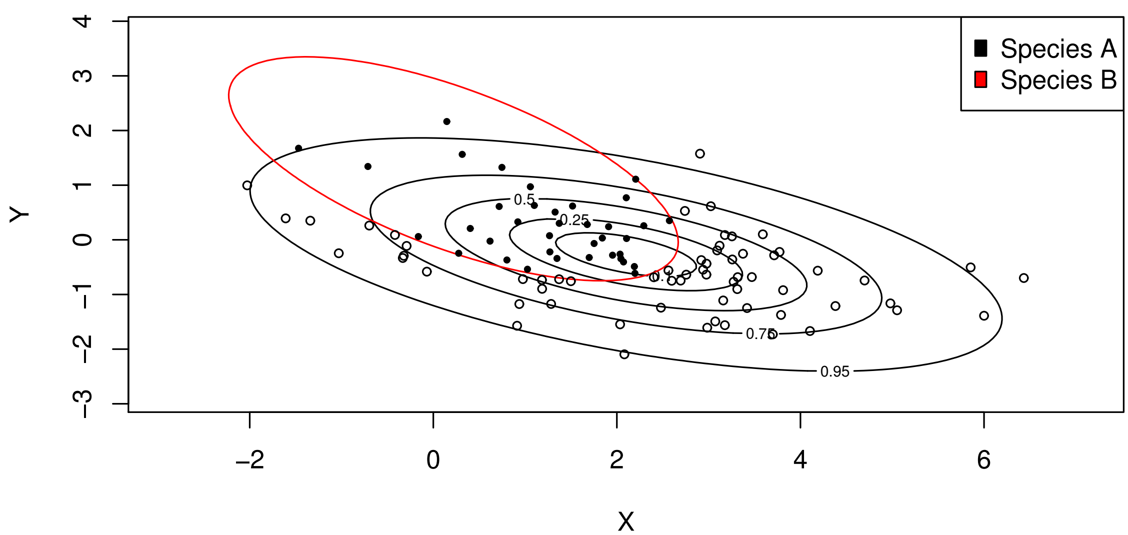

FIG. B1: Monte Carlo calculation of the probabilistic overlap of species A onto species B for a 2-d isotope distribution. The niche region of species B is plotted in red. The level curves of the isotope distribution in species A are plotted in black. $M = 100$ points are generated from the distribution of species A. Those that fall within $\mathrm{N_R}(B)$ are plotted with dots ($I_m = 1$) and those that do not are plotted with circles ($I_m = 0$). Since 37 points from this simulation are found within $\mathrm{N_R}(B)$, the Monte Carlo estimate of the overlap metric is $\hat{\mathrm{O}}\binom{A}{B} = 0.37$ with estimated standard error $\sqrt{.37 \cdot (1 - .37)/100} = .048$. For $M = 1$ million samples, the estimate of the overlap metric is $\hat{\mathrm{O}}\binom{A}{B} = 0.438$ with estimated standard error 4.96×10^{-4}.